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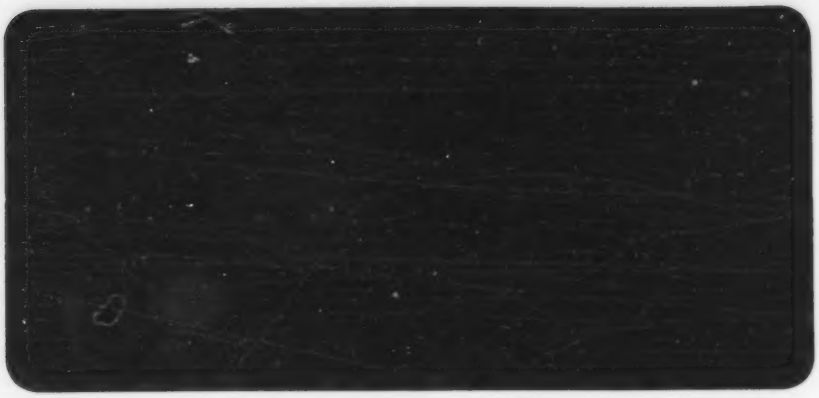

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The Dynamic Effects of Taxes and Government Spending in a Calibrated Canadian Endogenous Growth Model

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I. Introduction

The relationship between government spending, taxation, and economic growth has been one of the most important issues in economics. Unlike the traditional neo-classical growth theory which predicts no direct effects of economic policies on long-run economic growth, recent endogenous growth models of government spending and taxation, however, imply that fiscal policies may influence growth rates through their effects on real rates of return that affect the accumulation and allocation of both physical and human capital.

Taxation impacts on growth generally depend on whether tax revenue is spent productively or unproductively. This suggests that the assessment of particular tax increases or tax cuts should take into account the type of spending to be financed or reduced by the measure. Most studies on the growth effects of taxation, however, do not consider any productive use of tax revenue (for example, Rebelo (1991), Pecorino (1995), and Bull (1993)). Such studies conclude that taxation has negative impacts on growth. In contrast, Jones, Manuelli and Rossi (1993) show that, in a one-sector endogenous growth model in which government spending has direct positive effects on investment, the asymptotic tax rate on capital income is strictly positive.

Much of the early endogenous growth literature was purely theoretical and could not quantify the effects of taxes on growth in observed economies. Recently the computable general equilibrium modelling tools developed in the neo-classical framework have begun to be applied in an endogenous growth context (for example, King and Rebelo (1990)).

Many endogenous growth models are restricted to the analysis of steady-states with all the variables in the economy growing at their long-run rates. However, as Mulligan and Sala-I-Martin (1993) point out, if there are initial imbalances among the different sectors there may be a transitional period where the relevant variables do not behave as predicted by the steady-state analysis. Since the technologies of producing physical and human capital are usually different, the ratio of human to physical capital approaches a constant value only gradually as the economy approaches its balanced growth path. Moreover, the initial physical to human capital ratio or private to public capital ratios may not be the same as in the steady-state because of some recent shocks such as a tax rate change or a government spending cut.

A few studies explicitly compute transitional effects of taxes. Jones, Manuelli and Rossi (1993) derive optimal Ramsey tax paths in endogenous growth models and provide a quantitative estimate of the growth and welfare effects of moving from the current tax system to the optimal paths. Devereux and Love (1994) analyse the dynamic effects of

factor income and consumption taxes in a two-sector endogenous growth model. Their tax experiments involve increases in each tax. They find that while all forms of taxation reduce the growth rate, the particular dynamic response to each tax increase differs considerably among the different taxes. The response to a capital income tax is almost wholly in the form of substitution of factors out of final good production and into human capital production, while most of the adjustment in response to a wage or a consumption tax takes place via a fall in total hours worked in each sector, with no substantial reallocation of factors between sectors. These different tax effects on factor reallocation lead to different effects of tax regimes on the levels of output.

There are some important differences between this paper and those in the previous literature on taxation and government spending in endogenous growth models. First, different physical and human capital production technologies are considered, and both involve private as well as public investment. Modelling public investment in private production introduces second-round growth effects of government revenue and spending that are affected by tax changes. Second, transitional dynamics between steady-states are explicitly computed. Although the papers by Jones, Manuelli and Rossi (1993) and by Devereux and Love (1994) also compute transitional dynamics, they both miss some important features. The first paper considers the case of a simple one-sector model of capital accumulation that has no labour input when it endogenises government expenditure as an input in the production function. The second paper does not consider endogenous government expenditure at all. Third, this paper looks at different taxes and spending, and conducts different tax experiments from those in the existing literature. Three types of taxes and four types of government spending are analysed here: capital and labour income taxes and a sales tax; final good sector-specific and human capital sector-specific spending, pure public spending shared by both sectors, and household transfers. The experiments include 1 per cent of GDP per unit of human capital shifts from one tax to another, 1 per cent of GDP per unit of human capital cut in each tax accompanied by cuts in each type of spending, as well as cutting all spending types by the same proportion. Government budget is kept balanced in each experiment. Finally, the model is calibrated to the Canadian economy while the other models calibrate to U.S. data.

This paper develops a two-sector (final goods and human capital) general equilibrium endogenous growth model with government spending financed by the three distorting taxes. Endogenous labour-leisure choice is considered. The growth is driven by physical and human capital accumulation as well as public investment.

The importance of human capital accumulation and public investment in endogenous growth models has been emphasised by various studies. Schultz (1961) noted that the growth rate of output exceeded the growth rate of the relevant input measures (namely, employment and physical capital), suggesting that investment in human capital is probably the major explanation for this difference. Lucas (1988) and Becker, Murphy and Tamura (1990) point out that over half of output growth remains unexplained in the neo-classical growth model. An empirical study conducted by Tallman and Wang (1994)

claim that the accumulation of labour skill improvements, omitted from the measured labour input, is responsible for the apparent incompatibility between the input growth rates and the output growth rate.

The motivation for considering government spending as an input into private productions is that the private and public inputs may not be close substitutes (see Barro (1990)). If user fees are difficult to charge, private activities will not readily replace public activities. In some cases of human capital accumulation such as basic education, some external effects would cause private production to be too low without government support.

The model is calibrated to the 1994 Canadian economy, and used to analyse growth effects of balanced budget tax shifts and tax cuts accompanied by spending cuts.

The rest of the paper is organised as follows. Section II presents the model environment and derives the system of equations that generates steady-state growth rates. Section III examines and compares the growth effects of different tax policy regimes. Both the steady-states and the transitional dynamics are analysed. Finally, Section IV concludes and indicates further work.

II. The Model

2.1 The Set-Up

There are a constant large number of identical infinitely-lived agents in this economy. The representative agent seeks to maximise overall utility given by a constant intertemporal elasticity of substitution function:

$$U = \sum_{t=0}^{\infty}(1+\rho)^{-t}\frac{\left[C_t^{\omega} L_t^{1-\omega}\right]^{1-\sigma}}{1-\sigma} \tag{1}$$

where C_t is consumption per capita at time t, the only consumption good is measured in units of final output, and L_t is leisure activity. The parameter σ is the reciprocal of the intertemporal elasticity of substitution, ρ is the positive and constant rate of time preference, and ω is a utility function weight parameter that satisfies $0 < \omega < 1$.

The leisure activity is described by the following function:

$$L_t = l_t H_{t-1}^{\varepsilon} \tag{2}$$

where l_t is the agent's raw time of leisure and H_{t-1} is his human capital stock accumulated by the end of period t-1 that is available for period t activities. When $\varepsilon = 0$, the leisure activity is simply raw leisure time; when ε is positive, the leisure activity is quality time associated with human capital.

There are two sectors of production: human capital and final goods that are used for consumption and investment. Physical capital, human capital, and public inputs are used in the two productions. The public inputs include pure public goods available to both sectors and government assistance to each sector. The concept of human capital follows Lucas (1988). An individual's human capital represents his general skill level. More specifically, a worker with human capital H_t is the productive equivalent of two workers with $0.5H_t$ each, or a half-time worker with $2H_t$.[1] Human capital production involves activities such as education and training.

Each agent has one unit of time in each period. The representative agent chooses consumption paths of goods and leisure, as well as investment so as to maximise his life-time utility given by (1), subject to his budget constraint and capital accumulation constraints:

$$(1+\tau_t^s)(C_t + I_t) + P_t I_t^H = r_t K_{t-1} - \tau_t^r (r_t - \delta_k) K_{t-1} + (1-\tau_t^w) w_t (1-l_t) H_{t-1} + TR_t + \pi_t \quad (3)$$

$$K_t = I_t + (1-\delta_k) K_{t-1} \quad (4)$$

$$H_t = I_t^H + (1-\delta_H) H_{t-1} \quad (5)$$

where τ_t^s, τ_t^r, and τ_t^w are sales, capital income and wage tax rates in period t, P_t is the relative price of the human capital in terms of the final goods, r_t and w_t are gross of tax real rates of return to physical and human capital, TR_t is government transfers, and π_t denotes profits that the household receives from producers. The depreciation rates of physical and human capital δ_k and δ_H are assumed to be constant over time. Physical and human capital investments in period t are denoted as I_t and I_t^H, respectively.

The utility maximisation problem implies that

$$\left(\frac{C_{t+1}}{C_t}\right)^{1-\omega(1-\sigma)} = \frac{1}{1+\rho}\left[\frac{(1-\tau_{t+1}^r) r_{t+1} + \tau_{t+1}^r \delta_k + (1+\tau_{t+1}^s)(1-\delta_k)}{(1+\tau_{t+1}^s)}\right]\left(\frac{l_{t+1} H_t^e}{l_t H_{t-1}^e}\right)^{(1-\sigma)(1-\omega)} \quad (6)$$

Therefore, the growth rate of consumption in period t+1 rises with a higher rate of return to capital in the same period, and falls with larger values of τ_{t+1}^r, τ_{t+1}^s, δ_k, and ρ. It is also positively related to the growth of the leisure activity.

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[1] This concept of human capital follows Lucas (1988).

There are four components of government spending in each period: human capital sector-specific spending, final good sector-specific spending, public services and infrastructures, and transfers. At the end of period t, the government levies sales, capital income and wage taxes. The tax revenue finances transfers to the households and investments into the three types of government capital stocks which will be inputs of the productions at t+1.

The government budget constraint at t is given by:

$$I_{G_1 t} + I_{G_H t} + I_{Gt} + TR_t = \tau_t^s (C_t + I_t) + \tau_t^r (r_t - \delta_k) K_{t-1} + \tau_t^w w_t (1 - l_t) H_{t-1} \equiv GR_t \tag{7}$$

where GR_t denotes total government tax revenue, $I_{G_1 t}$, $I_{G_H t}$ and I_{Gt} are government investments in its stocks of final goods sector-specific and human capital sector-specific spending, and pure public spending, respectively.

Define $I_{G_1 t} = \lambda_{1t} GR_t$, $I_{G_H t} = \lambda_{Ht} GR_t$, and $I_{Gt} = \lambda_{Gt} GR_t$, where $0 \le \lambda_{1t} < 1$, $0 \le \lambda_{Ht} < 1$, $0 \le \lambda_{Gt} < 1$, and $0 \le \lambda_{1t} + \lambda_{Ht} + \lambda_{Gt} \le 1$.

The laws of motion of the three types of government capital are as follows:

$$G_{1t} = I_{G_1 t} + (1 - \delta_{G_1}) G_{1t-1} \tag{8}$$

$$G_{Ht} = I_{G_H t} + (1 - \delta_{G_H}) G_{Ht-1} \tag{9}$$

$$G_t = I_{Gt} + (1 - \delta_G) G_{t-1} \tag{10}$$

where δ_{G_1}, δ_{G_H}, and δ_G are the constant depreciation rates of the corresponding capital stocks.

The sequence of events can be described as follows: at the beginning of each period t, the representative household supplies his capital stock to the producers, decides the amount of time to spend on his leisure activity, and supplies the rest to final goods production and human capital accumulation. The representative final goods producer receives G_{1t-1} and G_{t-1} from the government as inputs into the production. He also rents K_{1t-1} units of physical capital and H_{1t-1} units of effective labour generated by N_t hours of work undertaken by a worker with H_{t-1} units of human capital. Similarly, the representative human capital producer receives G_{Ht-1} and G_{t-1} from the government, and rents K_{Ht-1} units of physical capital and H_{Ht-1} units of effective labour to produce new human capital. As the outputs of the two sectors are produced, the household receives his capital and labour income and profits. The household then pays taxes and receives government transfers TR_t. His after-tax net income will be used for his consumption C_t, physical

capital investment I_t, and human capital investment I_t^H. The physical and human capital stocks accumulated by the end of period t (K_t and H_t) are used in the productions next period.

All markets are assumed to be perfectly competitive. The representative producer in each sector makes his production decisions to maximise profits.

We define $K_{It-1} = \phi_t K_{t-1}$, $K_{Ht-1} = (1-\phi_t)K_{t-1}$, $H_{It-1} = N_t H_{t-1}$, and $H_{Ht-1} = (1-l_t-N_t)H_{t-1}$. Therefore, ϕ_t is the market proportion of total physical capital stock used to produce final goods in period t; and N_t is the hours spent on the final good production.

The technology of the human capital sector is:

$$Y_t^H = A_H K_{Ht-1}^{1-\alpha-\beta}(G_{Ht-1}^{\xi} G_{t-1}^{1-\xi})^{\alpha} H_{Ht-1}^{\beta} \tag{11}$$

where A_H is a technology parameter, α is the coefficient of total public inputs in the human capital production, and β is the coefficient of human capital input.

The production function of the final goods sector is:

$$Y_t^I = A_I K_{It-1}^{1-\eta-\gamma}(G_{It-1}^{\theta} G_{t-1}^{1-\theta})^{\eta} H_{It-1}^{\gamma} \tag{12}$$

where A_I is a technology parameter, η is the coefficient of total public inputs and γ is the coefficient of human capital input in the final goods production. The final output is defined as GDP.

When the physical capital stock and the available efficiency units of labour are optimally allocated between the two sectors, the marginal products of physical and human capital measured in terms of final goods are equal between them:

$$P_t A_H(1-\alpha-\beta)K_{Ht-1}^{-\alpha-\beta}(G_{Ht-1}^{\xi} G_{t-1}^{\xi})^{\alpha} H_{Ht-1}^{\beta} = A_I(1-\eta-\gamma)K_{It-1}^{-\eta-\gamma}(G_{It-1}^{\theta} G_{t-1}^{1-\theta})^{\eta} H_{It-1}^{\gamma} \tag{13}$$

and

$$P_t A_H \beta K_{Ht-1}^{1-\alpha-\beta}(G_{Ht-1}^{\xi} G_{t-1}^{1-\xi})^{\alpha} H_{Ht-1}^{\beta-1} = A_I \gamma K_{It-1}^{1-\eta-\gamma}(G_{It-1}^{\theta} G_{t-1}^{1-\theta})^{\eta} H_{It-1}^{\gamma-1} \tag{14}$$

Therefore, the relative price of human capital P_t equals the ratio of the marginal product of physical capital in the final goods sector to its marginal product in the human capital sector (implied by equation (13)). It also equals the ratio of the marginal product of human capital in the final goods sector to its marginal product in the human capital sector (implied by equation (14)).

Eliminating P_t from the two equations yields the following relation between the capital-effective labour intensities in the two sectors:

$$\frac{\beta(1-\phi_t)K_{t-1}}{(1-\alpha-\beta)(1-L-N_t)H_{t-1}} = \frac{\gamma\phi_t K_{t-1}}{(1-\eta-\gamma)N_t H_{t-1}} \tag{15}$$

Since an alternative to investing in one more unit of physical capital is to accumulate $1/P_t$ units of human capital, the following condition guarantees the optimal decision of investing in physical versus human capital:

$$\frac{(1-\tau^r_{t+1})r_{t+1} + (1+\tau^s_{t+1})(1-\delta_k)}{1+\tau^s_t} = \frac{(1-\tau^w_{t+1})w_{t+1}(1-l_{t+1}) + P_{t+1}(1-\delta_H) + (1+\tau^s_{t+1})(1-\omega)\varepsilon C_{t+1}/(\omega H_t)}{P_t} \tag{16}$$

This simply says that the net rate of return to one unit of physical capital equals the net rate of return to $1/P_t$ units of human capital.

The market clearing conditions are:

$$Y^I_t = C_t + I_t + I_{Gt} + I_{G_K t} + I_{G_H t} \tag{17}$$

$$K_t = K_{It} + K_{Ht} \tag{18}$$

$$H_t = H_{It} + H_{Ht} \tag{19}$$

2.2 State-Like and Control-Like Variables

It is convenient to rewrite state and control variables as variables that remain stationary when the levels of output and the capital stocks grow at a positive rate forever. This can be done by defining what Mulligan (1991) calls state-like and control-like variables. State-like variables are transformations of state variables only, with the property that, unlike the state variables H_t, K_t, G_{It}, G_{Ht} and G_t, they remain constant in the steady-state. In particular, we define:

$$\overline{K}_t \equiv K_t / H_t \ , \ \overline{K}_{It} \equiv K_{It} / H_t \ , \ \overline{K}_{Ht} \equiv K_{Ht} / H_t \tag{20}$$

$$\overline{G}_t \equiv G_t / H_t \ , \ \overline{G}_{It} \equiv G_{It} / H_t \ , \ \overline{G}_{Ht} \equiv G_{Ht} / H_t \tag{21}$$

We will define control-like variables as transformations of control variables that will not grow in the steady-state. In particular, we define:

$$\overline{C}_t \equiv C_t / H_t \ , \ \overline{Y}^I_t \equiv Y^I_t / H_t \ , \ \overline{Y}^H_t \equiv Y^H_t / H_t \tag{22}$$

Define $H_t / H_{t-1} = 1 + g_t$, where g_t denotes the growth rate of human capital in period t.

With the above normalisation, the first-order conditions of the household's optimisation problem and those of the producers, the government's budget constraint and laws of motion of its capital stocks, together with the goods and capital market clearing conditions, imply the following equilibrium conditions:

$$\overline{C}_t = \overline{C}_{t+1}(1+g_{t+1})\left\{\frac{(1+\rho)(1+\tau^s_{t+1})}{(1-\tau^r_{t+1})r_{t+1}+\tau^r_{t+1}\delta_k+(1+\tau^s_{t+1})(1-\delta_k)}\left[\frac{l_t}{l_{t+1}(1+g_t)^{\varepsilon}}\right]^{(1-\sigma)(1-\omega)}\right\}^{\frac{1}{1-\omega(1-\sigma)}} \tag{E1}$$

$$l_t = \frac{(1+\tau^s_t)(1-\omega)\overline{C}_t(1+g_t)}{(1-\tau^w_t)w_t\omega} \tag{E2}$$

$$P_t = \frac{(1+\tau^s_t)[(1-\tau^w_{t+1})w_{t+1}(1-l_{t+1})+P_{t+1}(1-\delta_H)+(1+\tau^s_{t+1})(1-\omega)\varepsilon\overline{C}_{t+1}(1+g_{t+1})/\omega]}{(1-\tau^r_{t+1})r_{t+1}+\tau^r_{t+1}\delta_k+(1+\tau^s_{t+1})(1-\delta_k)} \tag{E3}$$

$$\overline{Y}^I_t = \frac{A_I(\overline{G}^{\theta}_{It-1}\overline{G}^{1-\theta}_{t-1})^{\eta}(\overline{K}_{It-1})^{1-\eta-\gamma}N_t^{\gamma}}{1+g_t} \tag{E4}$$

$$\phi_t = \frac{(1-\eta-\gamma)\beta(1-\phi_t)N_t}{\gamma(1-\alpha-\beta)(1-l_t-N_t)} \tag{E5}$$

$$N_t = \frac{\gamma\overline{Y}^I_t(1+g_t)}{w_t} \tag{E6}$$

$$g_t+\delta_H = A_H(\overline{G}^{\xi}_{Ht-1}\overline{G}^{1-\xi}_{t-1})^{\alpha}\overline{K}^{1-\alpha-\beta}_{Ht-1}(1-l_t-N_t)^{\beta} \tag{E7}$$

$$r_t = \frac{P_t(1-\alpha-\beta)(g_t+\delta_H)}{(1-\phi_t)\overline{K}_{t-1}} \tag{E8}$$

$$w_t = \frac{P_t\beta(g_t+\delta_H)}{1-l_t-N_t} \tag{E9}$$

$$\overline{GR}_t = \tau^s_t(\overline{C}_t+\overline{I}_t)+\frac{\tau^r_t(r_t-\delta_k)\overline{K}_{t-1}}{1+g_t}+\frac{\tau^w_t w_t(1-l_t)}{1+g_t} \tag{E10}$$

$$\overline{I}_{Gt} = \lambda_{Gt}\overline{GR}_t \tag{E11}$$

$$\overline{I}_{G_I t} = \lambda_{G_I t}\overline{GR}_t \tag{E12}$$

$$\overline{I}_{G_H t} = \lambda_{G_H t}\overline{GR}_t \tag{E13}$$

$$\overline{Y}_t^I = (1+\tau_t^s)(\overline{C}_t + \overline{I}_t) + \frac{\tau_t^r(r_t - \delta_k)\overline{K}_{t-1}}{1+g_t} + \frac{\tau_t^w w_t(1-l_t)}{1+g_t} - \overline{TR}_t \tag{E14}$$

2.3 The Steady-State

We define the steady-state as the state where all the variables grow at a common constant rate. In the steady-state, the allocations of physical capital and labour between the two sectors are assumed to be unchanged. In particular, in the steady-state, the following conditions hold:

$$g_{t+1} = g_t \equiv g \tag{23}$$

$$P_{t+1} = P_t \equiv P \tag{24}$$

$$N_{t+1} = N_t \equiv N \tag{25}$$

$$l_{t+1} = l_t \equiv l \tag{26}$$

$$\phi_{t+1} = \phi_t \equiv \phi \tag{27}$$

$$\overline{C}_{t+1} = \overline{C}_t \equiv \overline{C} \tag{28}$$

Equations similar to (28) hold for other variables including $\overline{I}$, $\overline{K}$, $\overline{G}$, $\overline{G}_I$, and $\overline{G}_H$.

All the parameters including λ_I, λ_H and λ_G are constant in the steady-state.

We can obtain a steady-state system of 14 equations by substituting the steady-state conditions into the above equilibrium system. The steady-state system can be solved for the steady-state growth rate, the rate of return to capital, wage rate, physical and human capital stocks, the relative price of human capital, household's investment and consumption of goods and leisure, the levels of the three government capital stocks, as well as allocations of physical capital and effective labour between the two sectors.

The irreversible nature of investment (i.e., $I_t \geq 0$) implies that the lowest feasible growth rate is $-\delta_k$ which implies no investment[2]. The steady-state growth rate is the larger number of the rate implied by the above system and $-\delta_k$.

III. Tax and Spending Policy Regimes and Economic Growth: Transitional Dynamics

In this section, we shall study the dynamic effects of balanced budget tax and spending changes. We are particularly interested in how the interest rate, the growth rate of GDP, consumption, physical and human capital behave along transitions.

The policy regimes considered here include shifts from one tax to another of 1 per cent of GDP per human capital ($\overline{GDP}$), and 1 per cent of $\overline{GDP}$ cut in each tax accompanied by each spending cut as well as cutting all the spending by the same proportion.

3.1 Parameter values

In order to apply the structure outlined above for fiscal policy simulation analysis, we choose particular values for the parameters appearing in the steady-state equations by calibrating the model to the 1994 Canadian economy as a base case steady-state growth path.

Taste and technology parameters are displayed in Table 1. It is assumed that goods consumption takes a weight of 2/3 in the utility function with the rest going to leisure activity. Government capital is assumed to depreciate at the same rate (0.07) as private physical capital. The share parameters of effective labour in both sectors are generally considered to be 2/3.

The remaining parameters are calibrated. These are the rate of time preference, the intertemporal elasticity of substitution, the human capital depreciation rate, the two sectors' productivities, and the share parameters α, η, θ and ξ. The calibration yields a steady-state growth rate of 2.64 percent and a gross rate of return to capital of 19 percent[3].

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[2] Equation (4) implies that $I_t = K_t - (1-\delta_k)K_{t-1}$. It can be rewritten as $\overline{I}_t = \overline{K}_t - \frac{(1-\delta_k)}{(1+g_t)}\overline{K}_{t-1}$. In the steady-state, $\overline{K}_t = \overline{K}_{t-1} \equiv \overline{K}$, therefore, $\overline{I}_t \geq 0$ implies that $g \geq -\delta_k$.

[3] The user cost of capital is estimated to be 18 percent in Canada.

Table 1. Taste and Technology Parameters

Variable	Canada
ρ, rate of time preference	0.012
$1/\sigma$, intertemporal elasticity of substitution	0.55
ε, quality leisure parameter	0.15
ω, utility function weight	2/3
A_I, productivity of the final good sector	0.34
A_H, productivity of the human capital sector	0.55
δ_K, depreciation rate of physical capital	0.07
δ_H, depreciation rate of human capital	0.055
δ_G, depreciation rate of government capital	0.07
γ, share parameter of effective labour in the final good sector	2/3
β, share parameter of effective labour in the human capital sector	2/3
α, coefficient of aggregated public capital in the final good sector	0.25
η, coefficient of aggregated public capital in the human capital sector	0.11
θ, coefficient of sector-specific public capital in the final good sector	0.55
ξ, coefficient of sector-specific public capital in the human capital sector	0.75

The calibration also yields a private consumption over GDP ratio of 0.67, and a private investment over GDP ratio of 0.14. These values are very close to the observed ratios in recent years. The steady state will be taken as a starting point, serving as a benchmark case for the following simulations.

Table 2 gives tax rates and percentages of government expenditures on different functions. The capital income tax rate is defined as personal and corporate taxes paid on capital income as a per cent of overall profits. The labour income tax rate is defined as the sum of wage income taxes and payroll taxes (social security contributions) as a per cent of total labour compensation. The sales tax rate is defined as value-added, sales, and excise taxes as a per cent of pre-tax expenditure. The 1994 Canadian tax rates on capital and labour income are 0.46 and 0.29, respectively. The sales tax rate in the same year is 0.10. Among the total government revenue, 4 percent is spent specifically in the final good sector and 19 percent in the human capital sector, 18 percent on pure public goods, and the remaining 59 percent is spent on the transfers.

Table 2. Benchmark Tax Rates and Proportions of Government Expenditures

Variable	Canada
τ_k, capital income tax rate	0.46
τ_n, labour income tax rate	0.29
τ_c, sales tax rate	0.10
λ_I, proportion of final goods sector-specific government spending	0.04
λ_H, proportion of human capital sector-specific government spending	0.19
λ_G, proportion of pure public goods spending	0.18

Data source: Tax rates are calculated by OECD using the methodology of Mendoza, Razin and Tesar (1994). Proportions of each type of government spending in total tax revenue are calculated by Xu (1996) using the 1994 Statistics Canada data of government expenditure by function.

3.2 Simulation Results

In all the following simulation exercises agents are informed about a permanent tax policy change at the beginning of period 2. When a distorting tax is announced to change, producers adjust their allocations of physical and human capital in the two sectors. This affects the outputs of final goods and new human capital. The input adjustment also affects both the gross and after-tax net rate of return to capital which influences the households' consumption and saving decisions. The after-tax net rate of return to capital is a key determinant of growth. The rate of return to capital can be further affected by the productive government spending changes.

Detailed transitional dynamics of some interesting variables such as resource allocation between sectors, growth rate of each sector, the rates of returns to physical and human capital, the ratio of private investment to human capital, and the ratio of consumption to human capital are presented in Figures 1(A)-1(C) to Figures 15(A)-15(C) for corresponding cases. The following analysis explains and compares the transitional dynamics of the fifteen cases.

(1) 1 per cent of $\overline{GDP}$ shift from capital income tax to sales tax

Given the 29 percent wage tax, a 1 per cent of $\overline{GDP}$ shift from capital income tax to sales tax implies a new capital income tax rate at 39.3 percent and a new sales tax rate at 10.8 percent. The new rates calculated based on 1 percent of $\overline{GDP}_1$ are announced at the beginning of period 2 and become effective immediately.

A lower capital income tax encourages households to invest. The net impact of the tax shift on the households is a higher demand for final goods. In the short run, larger proportions of capital and effective labour move to final good production (see Figure 1(A)). Given the stocks of government capital which were determined at the end of period 1, the final good sector grows faster while the growth of new human capital slows down temporarily (see Figure 1(B)).

The gross rate of return to physical capital jumps up for one period and then drops down below the pre-shock steady-state level (see Figure 1(C)). The up-ward jump is caused by the reallocation of resources to final good production which results in more human capital in the production and raises the marginal product of physical capital. Households invest more so that private investment grows faster than human capital stock while consumption does not grow as fast as human capital (see Figure 1(C)). More final output and savings lead to a larger physical capital stock. As more capital is put into the final good production, the rate of return to capital starts to decrease. The investment/human capital ratio falls back towards the pre-shock steady-state level. As some productive resources move from the final goods to the human capital sector, the growth of human capital output picks up while the growth of final output slows down (see again Figure 1(B)).

Table 3. Long Run Real GDP Growth Rate Impacts of Alternative Balanced budget 1 per cent of $\overline{GDP}$ Tax/Expenditure Shifts (Percentage Point Difference)

Policy Change	year 10	year 30	new steady state
(1) shift from capital to sales taxes	*0.0344*	*0.0311*	*0.0302*
(2) shift from capital to labour taxes	*0.0065*	*0.0127*	*0.0116*
(3) shift from labour to sales taxes	*0.0412*	*0.0404*	*0.0406*
(4) cut in capital tax and pure public good spending	*0.0139*	*0.0126*	*0.0113*
(5) cut in capital tax and human capital spending	*0.0148*	*0.0037*	*0.0031*
(6) cut in capital tax and transfers	*0.0360*	*0.0406*	*0.0395*
(7) cut in capital tax and all public spending	*0.0267*	*0.0267*	*0.0256*
(8) cut in labour tax and pure public good spending	*0.0149*	*-0.0006*	*-0.0024*
(9) cut in labour tax and human capital spending	*0.0701*	*-0.0313*	*-0.0317*
(10) cut in labour tax and transfers	*0.0840*	*0.0773*	*0.0771*
(11) cut in labour tax and all public spending	*0.0588*	*0.0389*	*0.0371*
(12) cut in sales tax and pure public good spending	*-0.0307*	*-0.0424*	*-0.0444*
(13) cut in sales tax and human capital spending	*-0.0032*	*-0.0660*	*-0.0661*
(14) cut in sales tax and transfers	*0.0216*	*0.0219*	*0.0215*
(15) cut in sales tax and all public spending	*0.0037*	*-0.0074*	*-0.0089*

In the medium run, ϕ_t and N_t vary less. With increasing physical and human capital stocks, more of both types of capital are used in final good production. In the long run, ϕ_t and N_t become steady. The new steady-state growth rate of final output or GDP is 2.67 percent. This rate is slightly higher than the pre-shock steady-state growth rate 2.64 percent.

(2) 1 per cent of $\overline{GDP}$ shift from capital income tax to labour income tax

As in the above case, a lower capital income tax encourages households to invest and stimulates the growth of final output. However, a higher labour income tax has negative impact on future labour income and hence discourages labour supply and human capital accumulation (see Figure 2(A)). Human capital output decreases temporarily, showing stronger impact of the tax shift on human capital production (see Figure 2(B)). As less effective labour is moved into the final good production, the marginal product of capital decreases (see Figure 2(C)), and final output grows at a lower rate. The ratio of investment to human capital falls and the consumption to human capital ratio rises.

The higher long-run rate of return to effective labour and the lower long-run rate of return to capital attract some resources from the final good sector to the human capital sector, which stimulates human capital accumulation. As a result, the new steady-state growth rate is lower than the medium-term GDP growth rate but higher than the benchmark growth rate.

(3) 1 per cent of $\overline{GDP}$ shift from labour income tax to sales tax

A lower labour income tax leads to increased labour supply and human capital investment while a higher sales tax negatively affects the households' demand for final goods. The physical and human capital allocations between the sectors hence move in an opposite direction to the above two cases (see Figure 3(A)). In the short run, the final good sector grows at a lower rate while the human capital sector grows faster (see Figure 3(B)). A smaller fraction of human capital input in the final goods sector tends to reduce the marginal product of capital (see Figure 3(C)).

As the human capital stock grows faster, the increased labour supply and human capital stock raise the marginal product of physical capital which encourages investment. Final output thus grows faster. More rapid capital stock growth drives down its marginal product so that some resources move to human capital production, and final output growth eventually slows down slightly.

(4) 1 per cent of $\overline{GDP}$ cut in capital income tax, accompanied by a pure public spending cut

The short-run market reaction to the capital income tax cut is similar to case (1). Since the tax revenue generated in period 2 finances government spending in period 3, the negative impacts of the public spending cut on production appears in period 3 when

growth slows down. However, more investment encouraged by higher after-tax rate of return raises marginal product of labour, which attracts more labour supply. As a result, the GDP growth rate increases. For more details of the transitional dynamics, see Figures 4(A)-4(C).

(5) 1 per cent of $\overline{GDP}$ cut in capital income tax, accompanied by a human capital sector-specific spending cut

The transitional dynamics of resource allocations, the rates of returns to capital and effective labour, savings and consumption are similar to case (4). The human capital-sector specific spending cut does not immediately affect final goods production, but has negative impacts on the future production of final goods through its negative effects on the human capital accumulation. Therefore, in this case the GDP growth rate does not drop below the pre-shock steady-state level while in case (4) the rates in a few periods starting in period 3 are lower than the pre-shock level. Since human capital-specific spending has a large coefficient in the human capital production function, the long-run negative effects on growth are slightly stronger here than case (4). In particular, the new steady-state growth rate in this case is 0.0264 while in the above case it is 0.0265.

(6) 1 per cent of $\overline{GDP}$ cut in capital income tax, accompanied by a lump-sum transfer cut

The transitional dynamics are similar to cases (4) and (5), except that the increase in the new steady-state growth rate here is larger than the above two cases. The reason is that cutting the transfer has no negative impacts on private productions while cutting either of the other two types of spending negatively affects long-run growth.

(7) 1 per cent of $\overline{GDP}$ cut in capital income tax, accompanied by a same proportion cut in all the spending

The transitional dynamics are a mix of cases (4) to (6). In particular, after some periods of faster GDP growth, the positive impact on the growth of the tax cut supported by cutting transfers dominates the negative growth effects of the productive spending cut.

(8) 1 per cent of $\overline{GDP}$ cut in labour income tax, accompanied by a pure public spending cut

A lower labour income tax encourages more effective labour supply. Given limited hours available for work, this is achieved by increasing human capital investment. In the short run, as more effective labour is relocated from the final good production to the production of new human capital, the marginal product of capital decreases and private savings and investment grow relatively slower than the human capital stock. This and the pure public spending cut lead to lower GDP growth rates.

In the longer run, higher effective labour supply growth raises the marginal product of capital which boosts households' incentives to invest. More resources are used to produce the final goods, and GDP growth picks up. However, ultimately the rate of return to capital is below the benchmark level, and the new steady-state growth rate is slightly lower than the benchmark level.

(9) 1 per cent of $\overline{GDP}$ cut in labour income tax, accompanied by a human capital sector-specific spending cut

The transitional dynamics of resource allocations follow a similar path to case (8). However, the growth rate swings around the benchmark rate are larger in this case because of the human capital sector-specific spending cut. In particular, the spending has a relatively large coefficient in the human capital production, and the spending cut lowers the marginal products of private inputs in the human capital sector. In the long run, the net effect on growth of the tax and spending cut is negative and stronger than that in the above case. The new steady-state growth rate is 0.0261 here and 0.0264 in case (8).

(10) 1 per cent of $\overline{GDP}$ cut in labour income tax, accompanied by a lump-sum transfer cut

The higher labour supply after the tax cut is very stable along the transitional dynamics. In the above two cases increased labour supply falls back somehow in the longer run because the productive spending cuts negatively affect the marginal products of effective labour. In contrast, there is no such negative effect in this case simply because the transfer cut has no negative impact on the productions. The long-run growth rate is higher than the benchmark case.

(11) 1 per cent of $\overline{GDP}$ cut in labour income tax, accompanied by a same proportion cut in all the spending

The transitional dynamics here are a mix of cases (8) to (10). In particular, after some periods of faster GDP growth, the positive impact of the tax cut supported by cutting transfers on the growth still dominates the negative growth effects of the other spending cuts. The percentage point difference between the new and the benchmark steady-states is not as large as in case (10) where there is no productive spending cut.

(12) 1 per cent of $\overline{GDP}$ cut in sales tax, accompanied by a pure public spending cut

The public spending cut negatively affects the productions of final goods and new human capital. These negative effects dominate the positive growth effect of a lower sales tax. Therefore, lower GDP growth rates present along the transitional dynamics and in the new steady state.

(13) 1 per cent of $\overline{GDP}$ cut in sales tax, accompanied by a human capital sector-specific spending cut

The transitional dynamics here are similar to case (12). The magnitude differences are caused by different spending cut. Since the human capital sector-specific spending cut discourages human capital accumulation, it indirectly affects the GDP growth through its impact on the effective labour input in the final good production. In contrast to case (12) where the spending cut affects directly the final good production and the negative impact on human capital accumulation is not as strong, the growth decline here is initially relatively small but larger in the long run.

(14) 1 per cent of $\overline{GDP}$ cut in sales tax, accompanied by a lump-sum transfer cut

The transfer cut does not have any negative impacts of the other spending cuts. The result here is a faster GDP growth.

(15) 1 per cent of $\overline{GDP}$ cut in sales tax, accompanied by a same proportion cut in all the spending

Again, the transitional dynamics here are a mix of cases (12) to (14).

IV. Conclusions

This paper has developed a computable general equilibrium endogenous growth model with tax-financed government spending in the private sectors. The model is calibrated to the 1994 Canadian economy, and used to analyse the growth effects of a 1 per cent of $\overline{GDP}$ shift from one tax to another, and 1 per cent of $\overline{GDP}$ cuts in each tax accompanied by different spending cuts while keeping the government budget balanced.

The results show that a 1 per cent of $\overline{GDP}$ shift from a capital income tax to either a labour income tax or a sales tax has positive growth effects that are stronger in the latter case. A capital income tax cut accompanied by any spending cut has positive effects on growth although the effects are quantitatively different. While a labour income tax cut accompanied by reduced lump-sum household transfers increases growth, the same tax cut accompanied by a productive spending cut lowers growth. In the case of a sales tax cut, the qualitative growth effects are similar to the case of a labour income tax cut. However, the quantitative growth effects are different.

The model simulations suggest that capital income tax is more distorting than either wage or sales taxes. Sales tax is the least distorting. Such a ranking is consistent with the standard result in the neo-classical growth literature. This is not surprising given that the initial impetus for higher growth often stems from higher government revenue and spending driven by income gains arising from the standard neo-classical saving channel.

Beyond that, the results also underscore the importance of modelling the impacts of tax changes on expenditure. In particular, the revenue effects of tax changes and the allocation of that revenue can have important implications for the results, with spending reductions in the human capital sector having the most negative long-run growth effects.

The interaction of the tax and expenditure impacts on growth leads to quite small long-run growth effects of which most are less than 0.4 per cent per year. Since expenditure impacts have a time lag, transitional growth impacts can be larger.

It should be stressed that actual economies are very complex, comprising many structural details, linkages and engines of growth that the model does not presently capture. Therefore, its results are not sufficient to determine the optimal tax/expenditure mix along the growth path. Future work will assess the sensitivity of the model's results to different parameter values.

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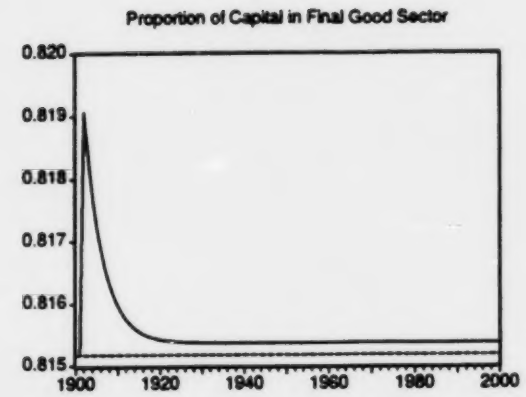
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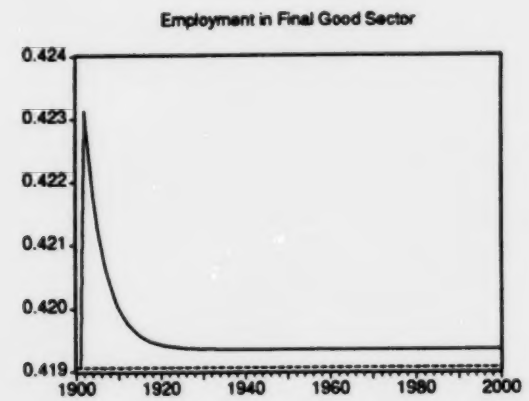
0.819
0.818
0.817
0.816
0.815
1900
1920
1940
1960
1980
2000


Employment in Final Good Sector
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0.423
0.422
0.421
0.420
0.419
1900
1920
1940
1960
1980
2000

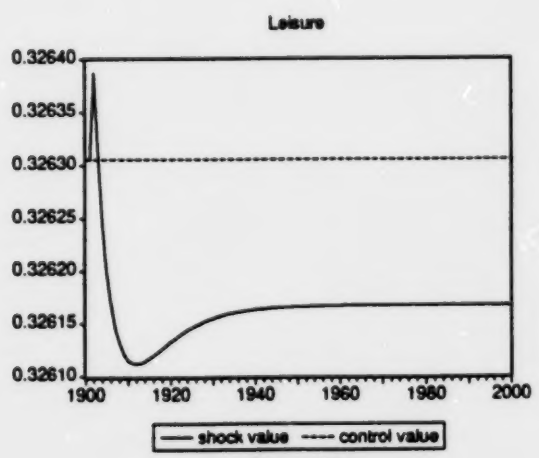
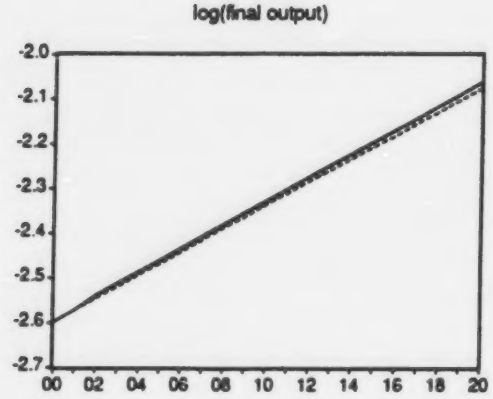
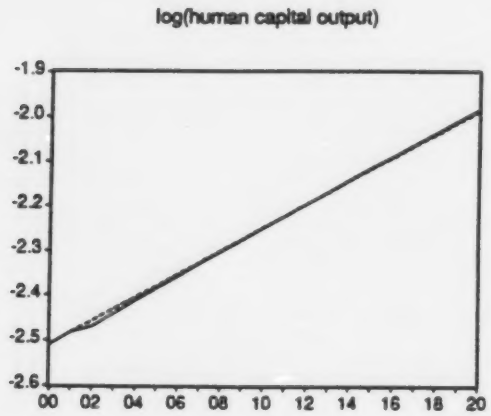

Leisure
0.32640
0.32635
0.32630
0.32625
0.32620
0.32615
0.32610
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 1(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

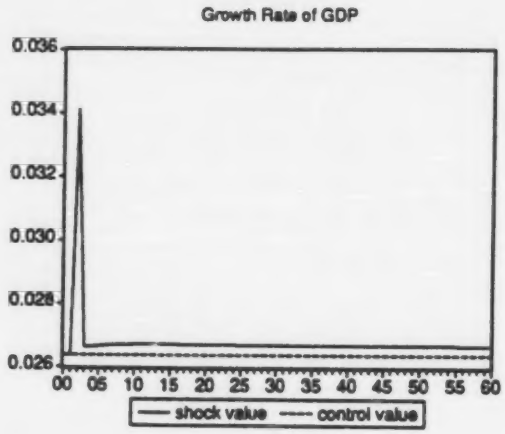

Growth Rate of GDP
0.036
0.034
0.032
0.030
0.028
0.026
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 1(B)

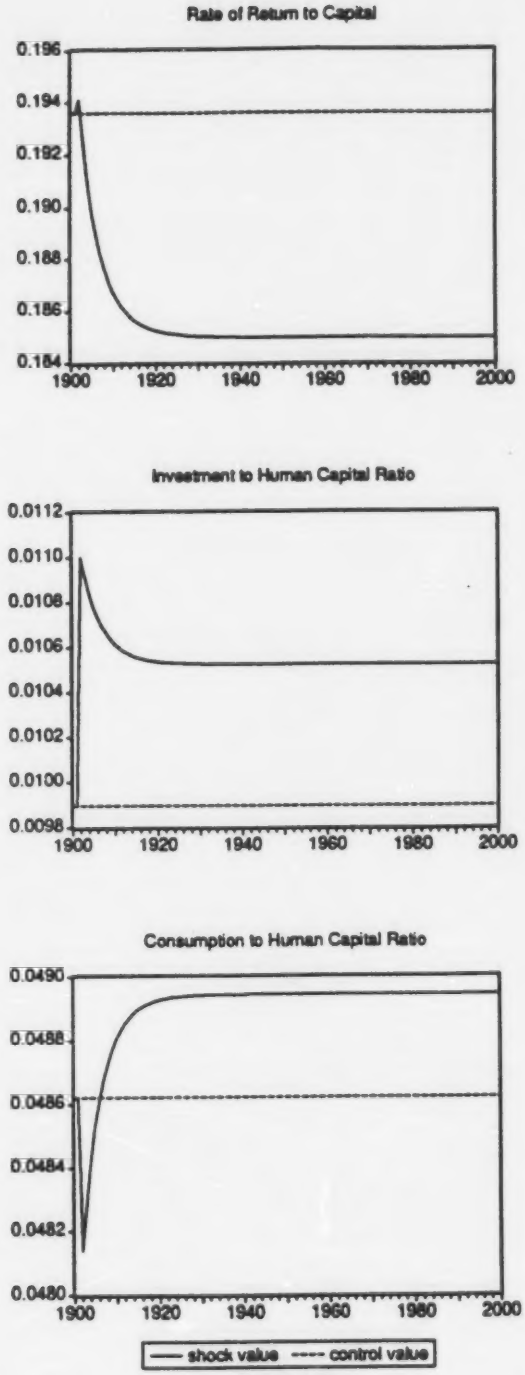
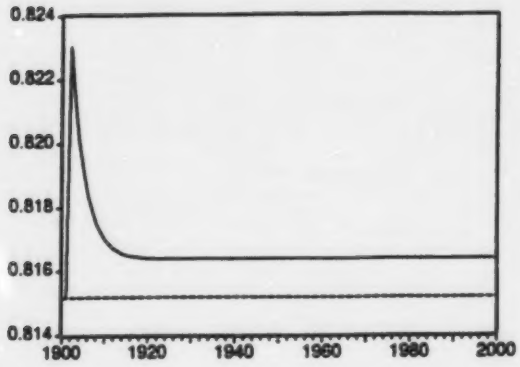
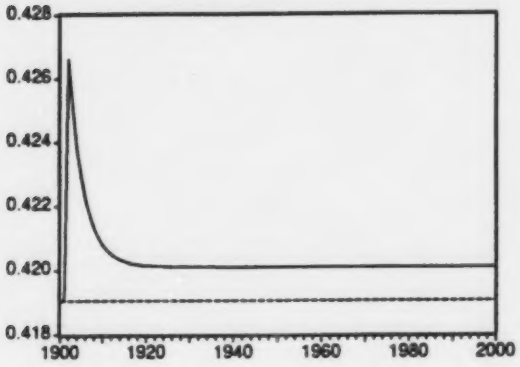

Rate of Return to Capital
0.196
0.194
0.192
0.190
0.188
0.186
0.184
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0112
0.0110
0.0108
0.0106
0.0104
0.0102
0.0100
0.0098
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0490
0.0488
0.0486
0.0484
0.0482
0.0480
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 1(C)

Propportion of Capital in Final Good Sector



Employment in Final Good Sector



Leisure

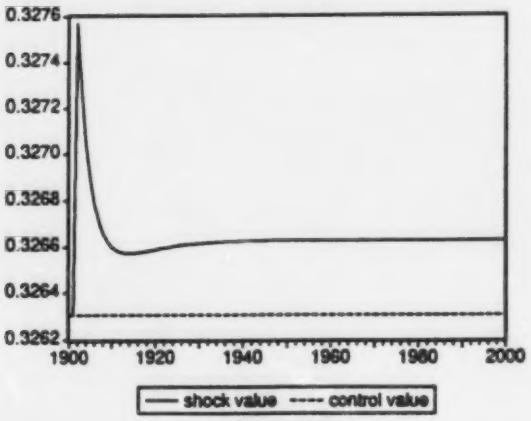


Figure 2(A)

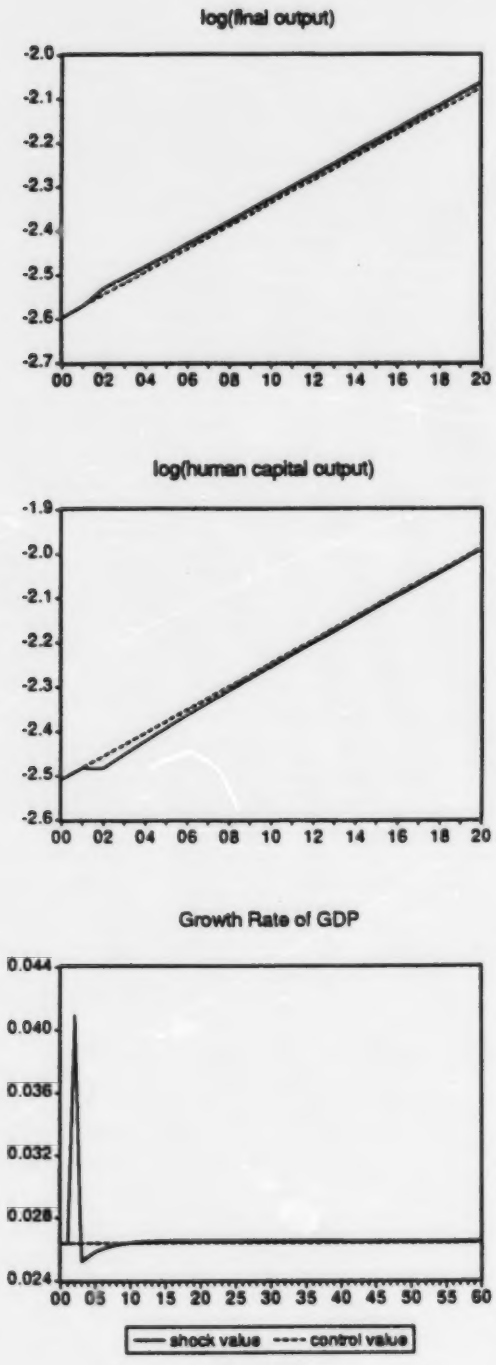
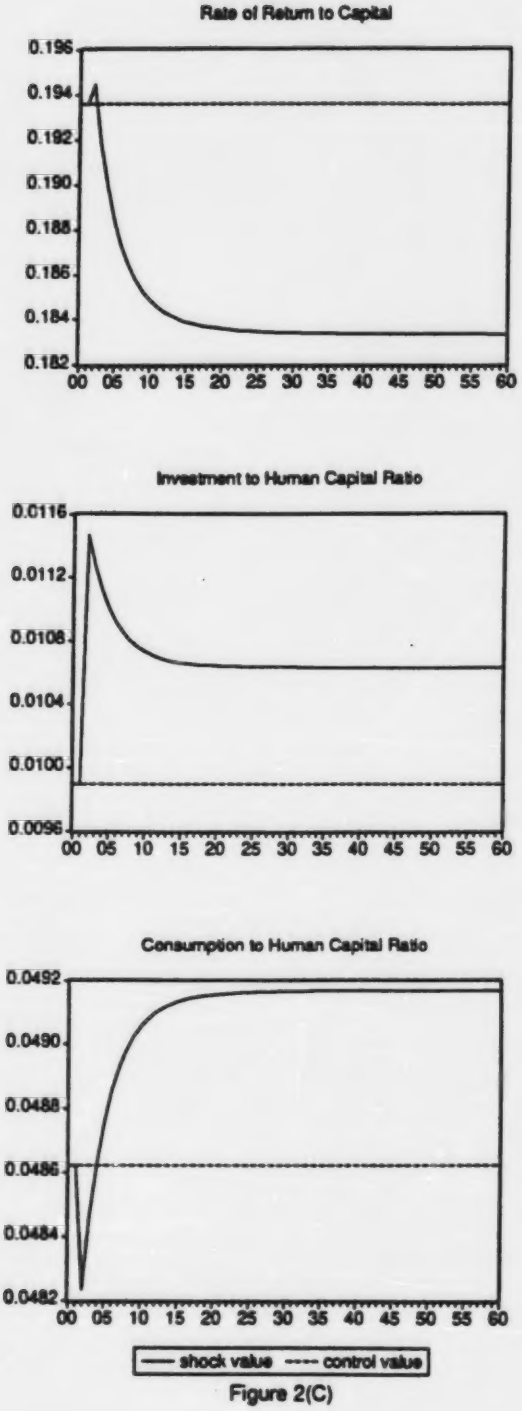

log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20
log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20
Growth Rate of GDP
0.044
0.040
0.036
0.032
0.028
0.024
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 2(B)


Rate of Return to Capital
0.196
0.194
0.192
0.190
0.188
0.186
0.184
0.182
00 05 10 15 20 25 30 35 40 45 50 55 60
Investment to Human Capital Ratio
0.0116
0.0112
0.0108
0.0104
0.0100
0.0096
00 05 10 15 20 25 30 35 40 45 50 55 60
Consumption to Human Capital Ratio
0.0492
0.0490
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0.0486
0.0484
0.0482
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 2(C)

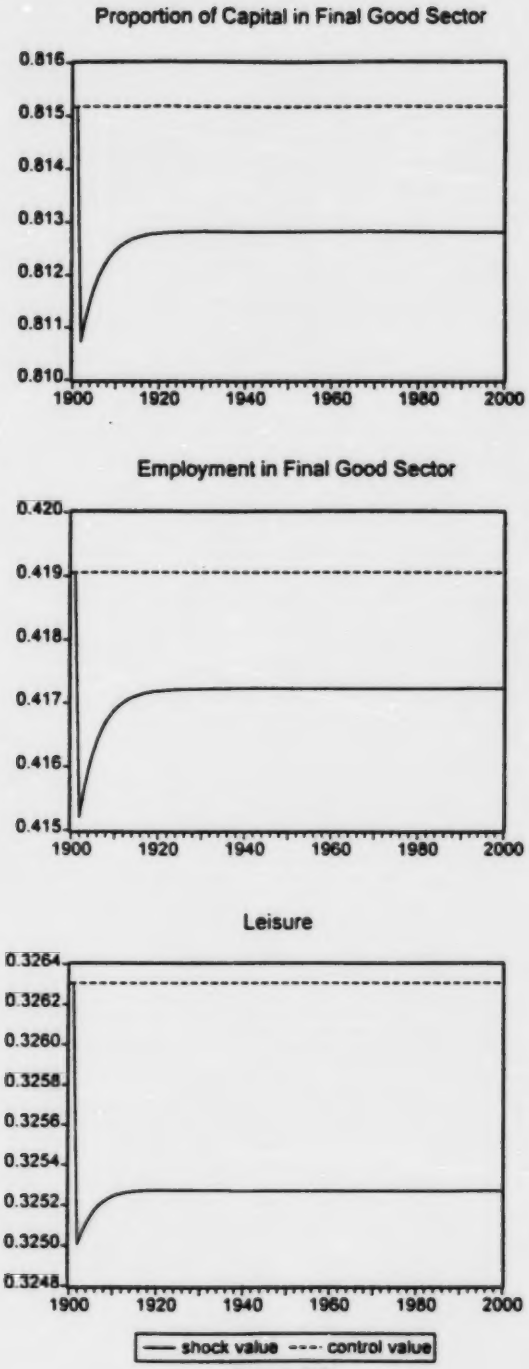
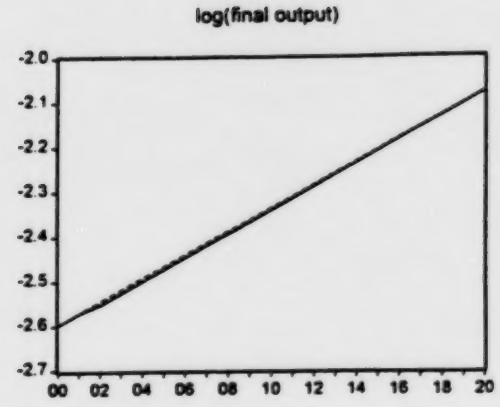
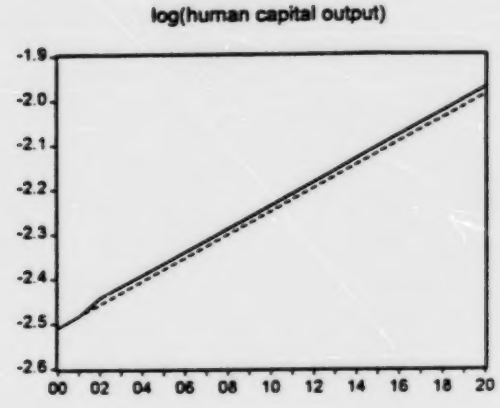

Proportion of Capital in Final Good Sector
0.816
0.815
0.814
0.813
0.812
0.811
0.810
1900
1920
1940
1960
1980
2000
Employment in Final Good Sector
0.420
0.419
0.418
0.417
0.416
0.415
1900
1920
1940
1960
1980
2000
Leisure
0.3264
0.3262
0.3260
0.3258
0.3256
0.3254
0.3252
0.3250
0.3248
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 3(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

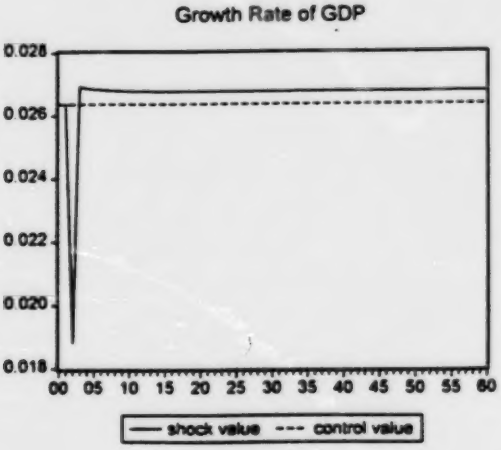
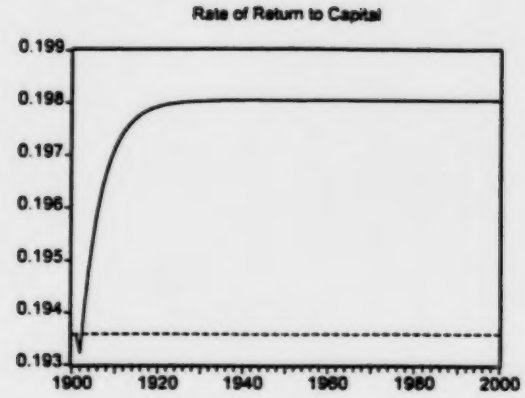
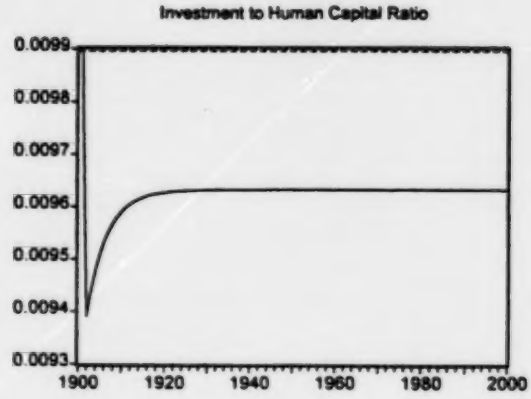

Growth Rate of GDP
0.028
0.026
0.024
0.022
0.020
0.018
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 3(B)


Rate of Return to Capital
0.199
0.198
0.197
0.196
0.195
0.194
0.193
1900
1920
1940
1960
1980
2000


Investment to Human Capital Ratio
0.0099
0.0098
0.0097
0.0096
0.0095
0.0094
0.0093
1900
1920
1940
1960
1980
2000

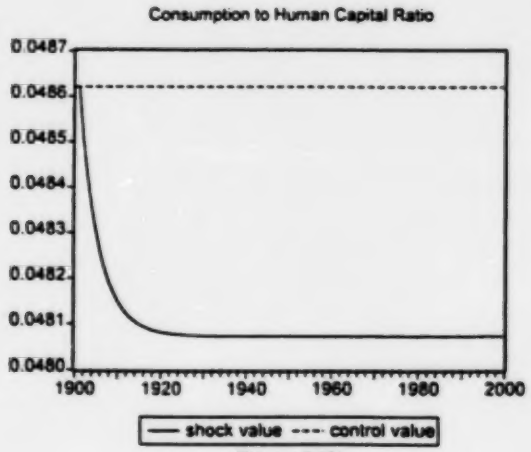
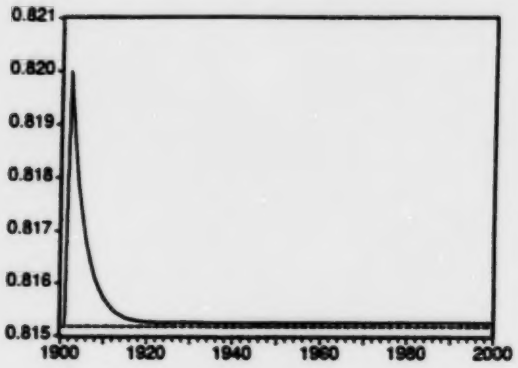
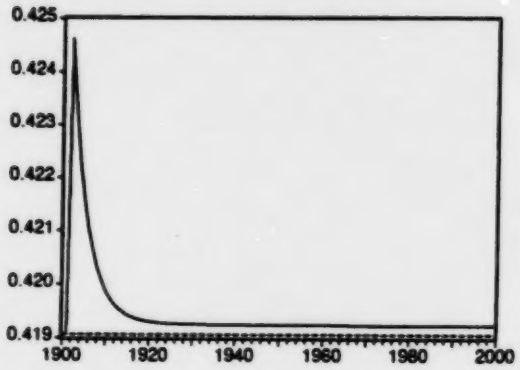

Consumption to Human Capital Ratio
0.0487
0.0486
0.0485
0.0484
0.0483
0.0482
0.0481
0.0480
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 3(C)

Proportion of Capital in Final Good Sector



Employment in Final Good Sector



Leisure

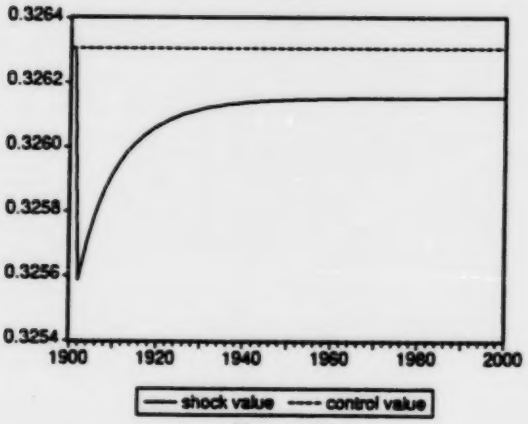
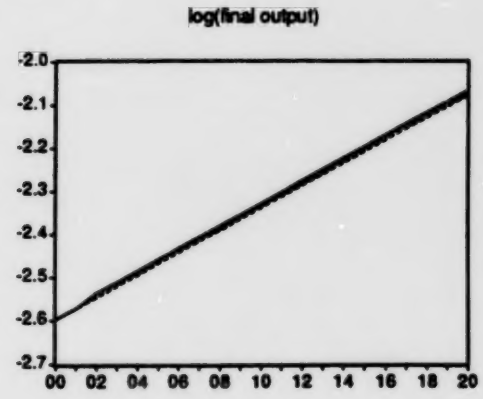
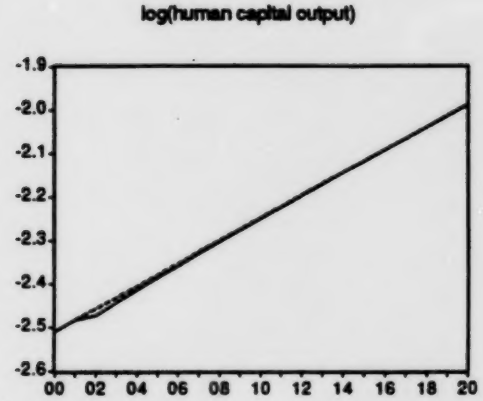


Figure 4(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

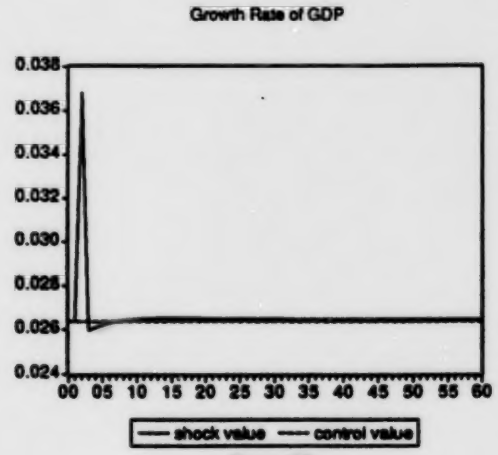

Growth Rate of GDP
0.038
0.036
0.034
0.032
0.030
0.028
0.026
0.024
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 4(B)

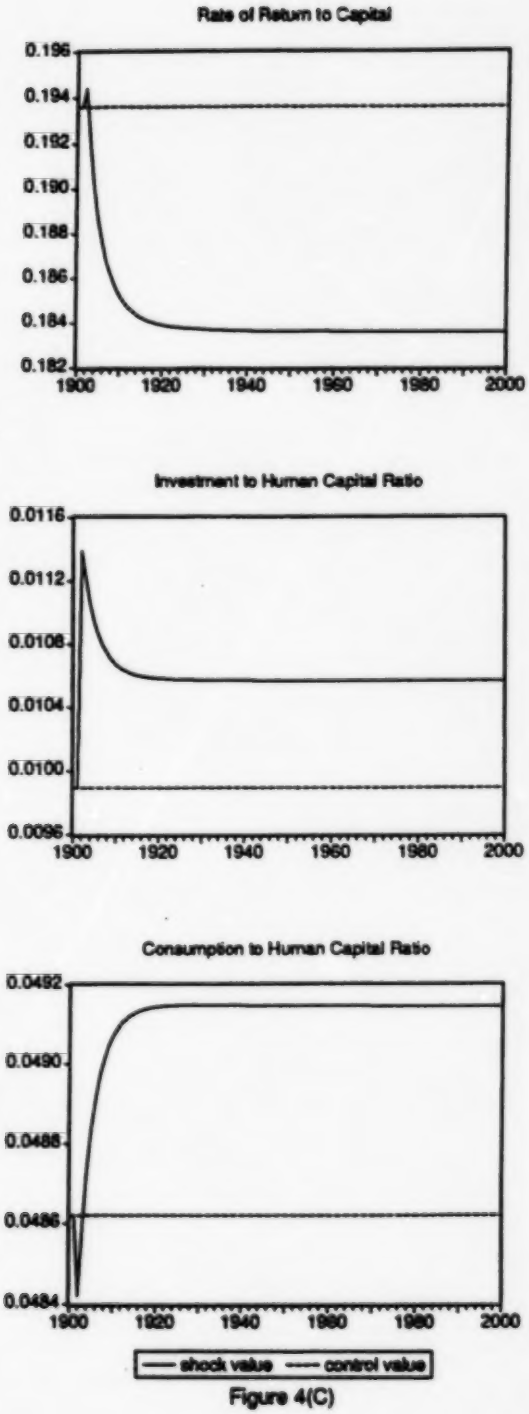
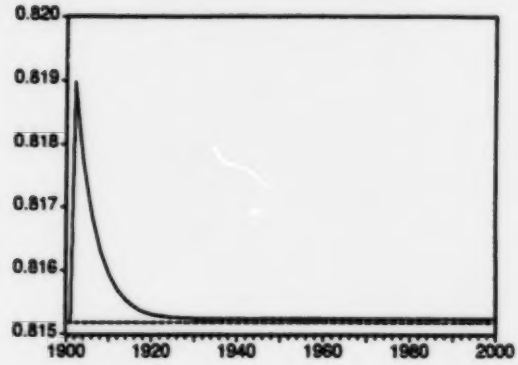
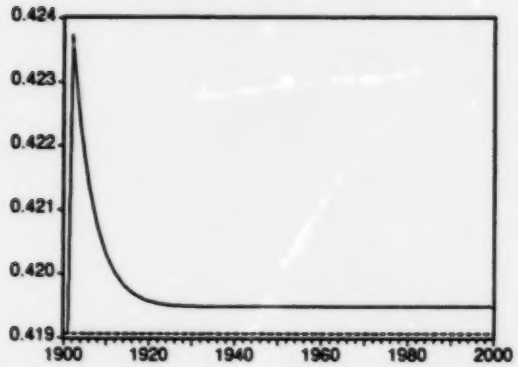

Rate of Return to Capital
0.196
0.194
0.192
0.190
0.188
0.186
0.184
0.182
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0116
0.0112
0.0108
0.0104
0.0100
0.0096
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0492
0.0490
0.0488
0.0486
0.0484
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 4(C)

Proportion of Capital in Final Good Sector



Employment in Final Good Sector



Leisure

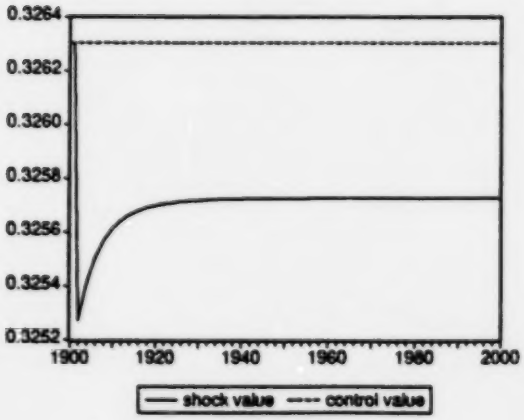
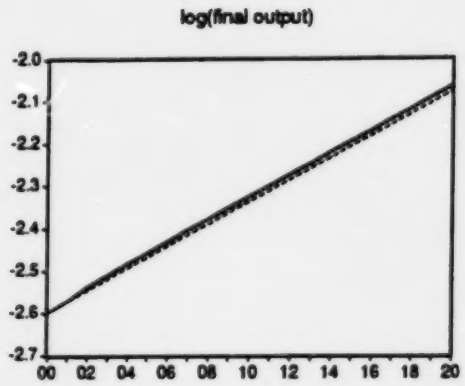
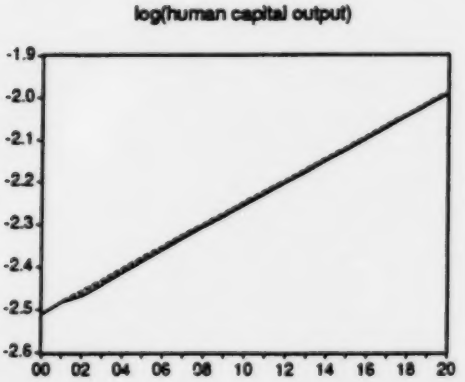


Figure 5(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

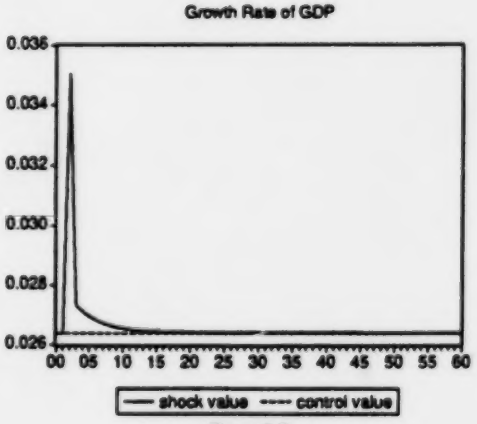

Growth Rate of GDP
0.036
0.034
0.032
0.030
0.028
0.026
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 5(B)

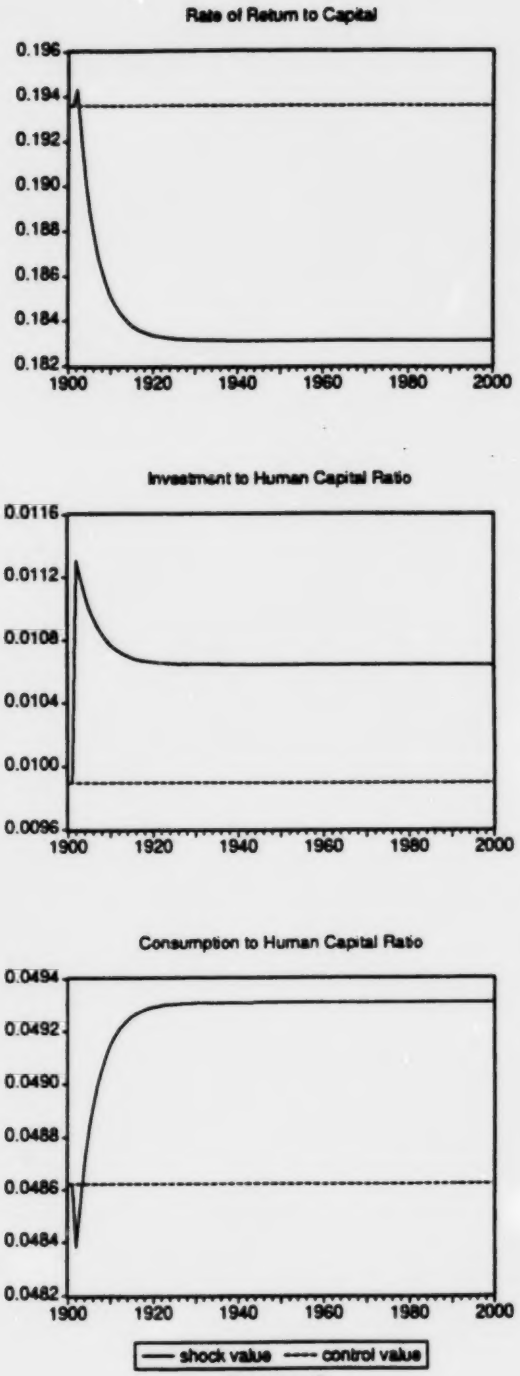
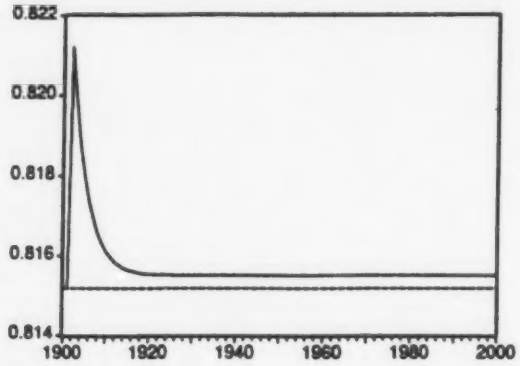
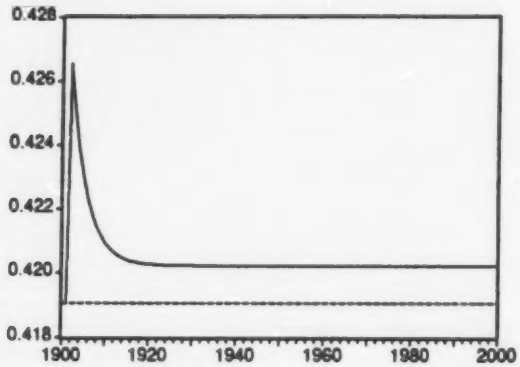

Rate of Return to Capital
0.196
0.194
0.192
0.190
0.188
0.186
0.184
0.182
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0116
0.0112
0.0108
0.0104
0.0100
0.0096
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0494
0.0492
0.0490
0.0488
0.0486
0.0484
0.0482
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 5(C)

Proportion of Capital in Final Good Sector



Employment in Final Good Sector



Leisure

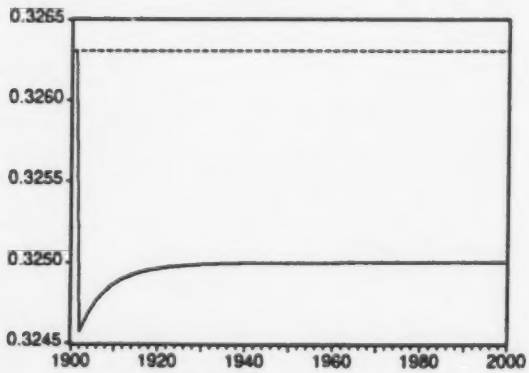
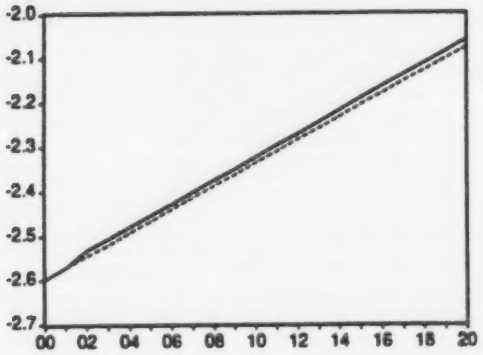
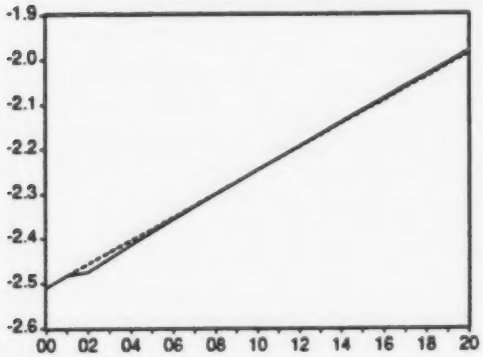


Figure 6(A)

log(final output)



log(human capital output)



Growth Rate of GDP

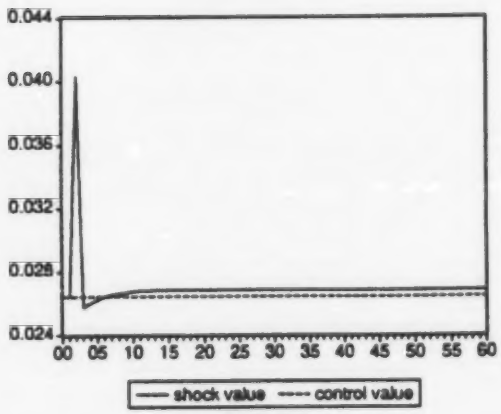
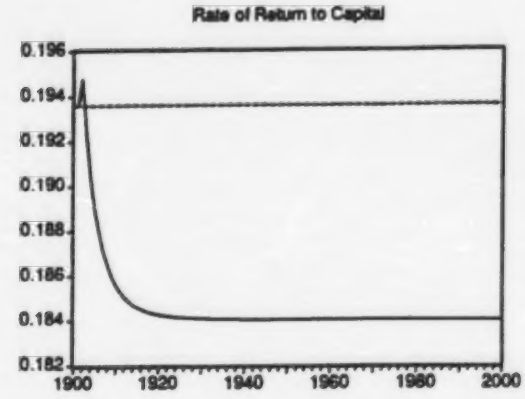
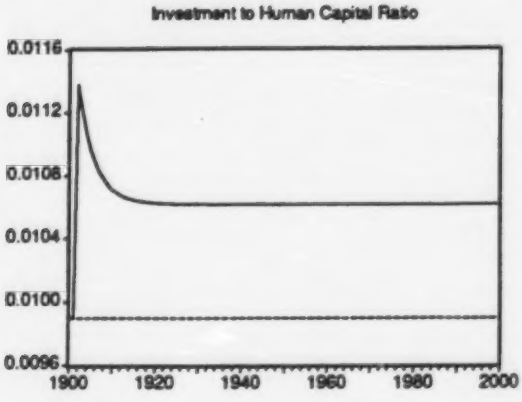


Figure 6(B)


Rate of Return to Capital
0.196
0.194
0.192
0.190
0.188
0.186
0.184
0.182
1900
1920
1940
1960
1980
2000


Investment to Human Capital Ratio
0.0116
0.0112
0.0108
0.0104
0.0100
0.0096
1900
1920
1940
1960
1980
2000

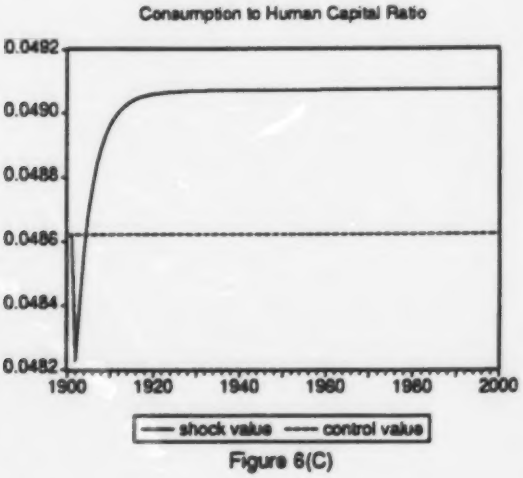

Consumption to Human Capital Ratio
0.0492
0.0490
0.0488
0.0486
0.0484
0.0482
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 6(C)

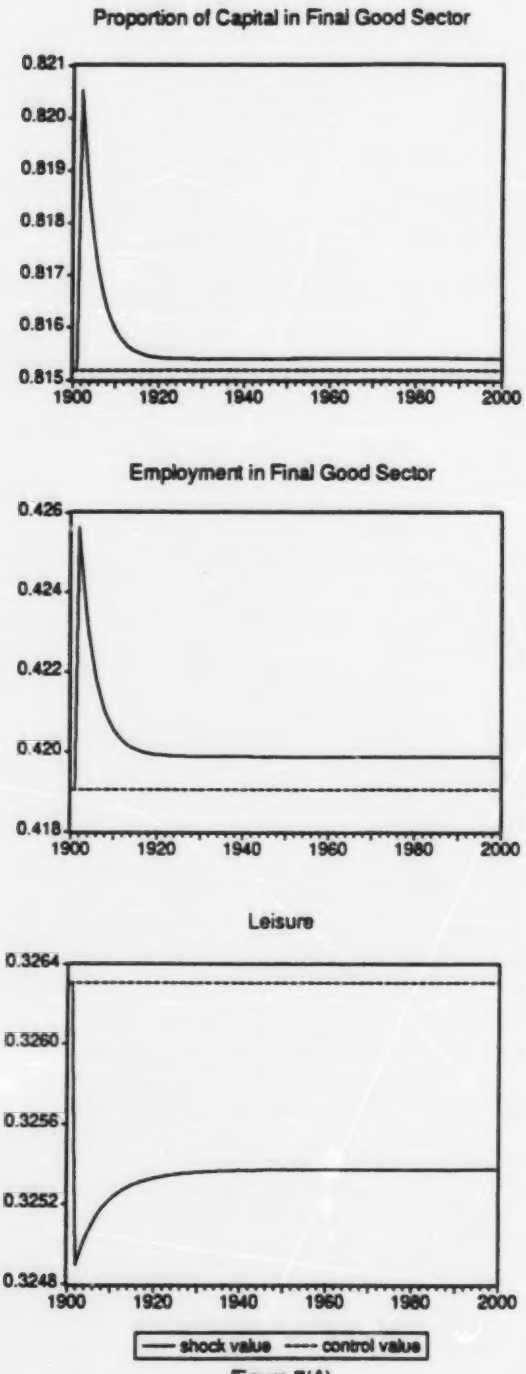
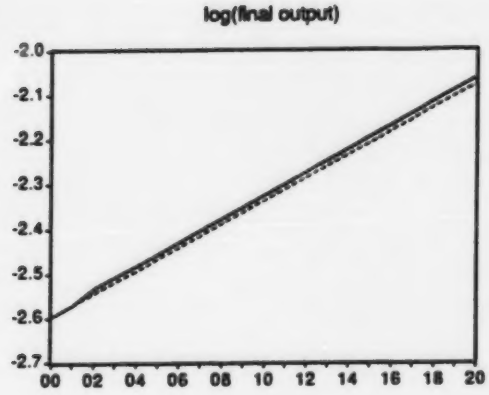
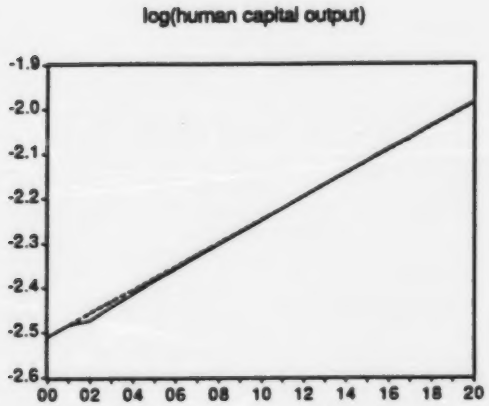

Proportion of Capital in Final Good Sector
0.821
0.820
0.819
0.818
0.817
0.816
0.815
1900
1920
1940
1960
1980
2000
Employment in Final Good Sector
0.426
0.424
0.422
0.420
0.418
1900
1920
1940
1960
1980
2000
Leisure
0.3264
0.3260
0.3256
0.3252
0.3248
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 7(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

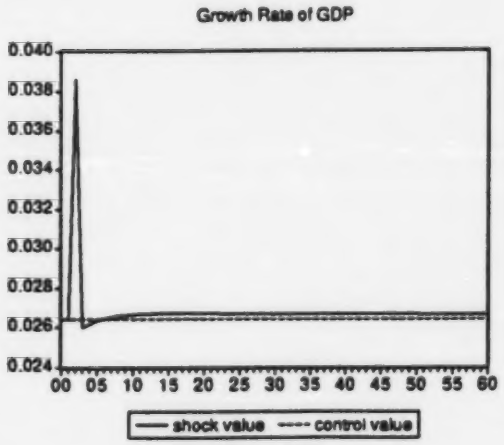

Growth Rate of GDP
0.040
0.038
0.036
0.034
0.032
0.030
0.028
0.026
0.024
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 7(B)

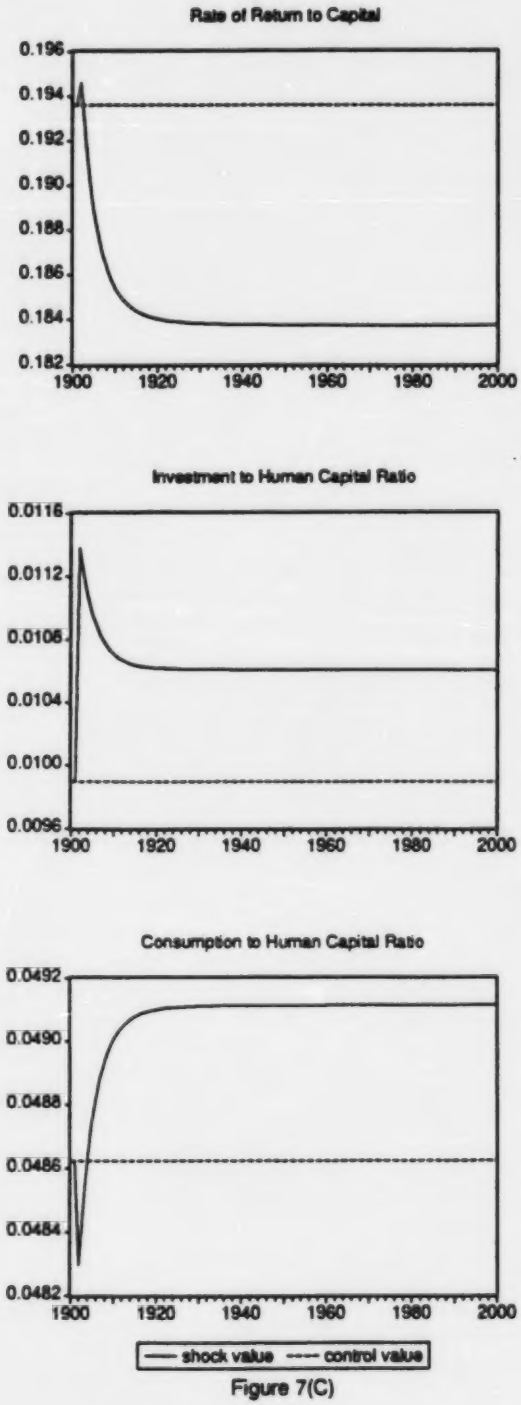
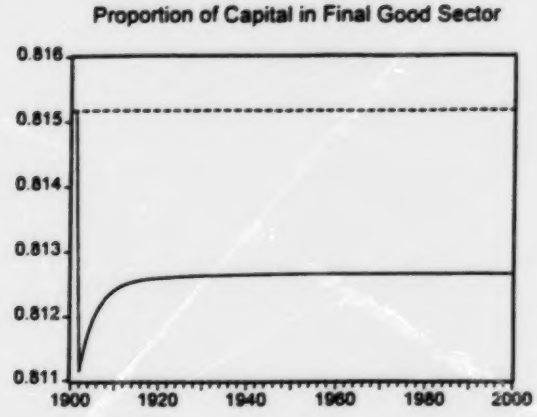

Rate of Return to Capital
0.196
0.194
0.192
0.190
0.188
0.186
0.184
0.182
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0116
0.0112
0.0108
0.0104
0.0100
0.0096
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0492
0.0490
0.0488
0.0486
0.0484
0.0482
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 7(C)


Proportion of Capital in Final Good Sector
0.816
0.815
0.814
0.813
0.812
0.811
1900
1920
1940
1960
1980
2000

Employment in Final Good Sector
0.420
0.419
0.418
0.417
0.416
0.415
1900
1920
1940
1960
1980
2000

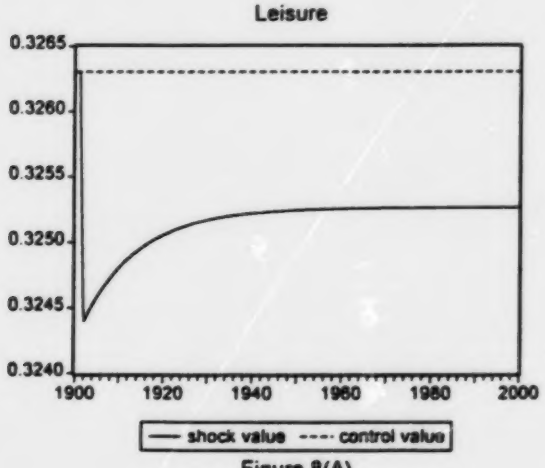
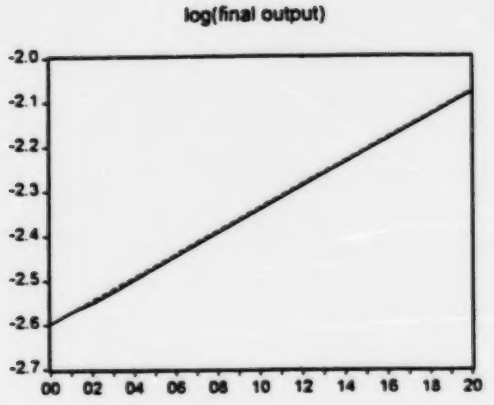
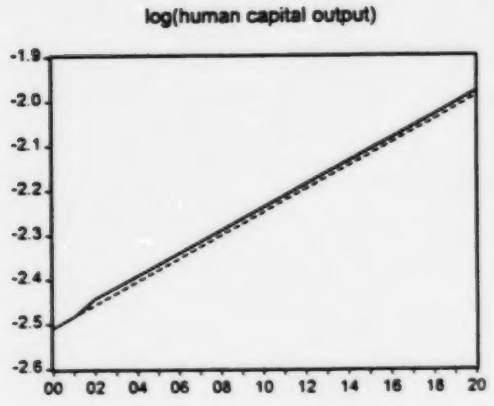

Leisure
0.3265
0.3260
0.3255
0.3250
0.3245
0.3240
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 8(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

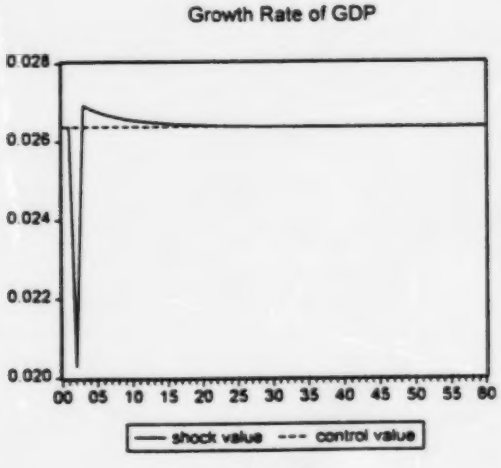

Growth Rate of GDP
0.028
0.026
0.024
0.022
0.020
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 8(B)

Rate of Return to Capital
0.1946
0.1944
0.1942
0.1940
0.1938
0.1936
0.1934
0.1932
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.00990
0.00985
0.00980
0.00975
0.00970
0.00965
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.04890
0.04885
0.04880
0.04875
0.04870
0.04865
0.04860
0.04855
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 8(C)

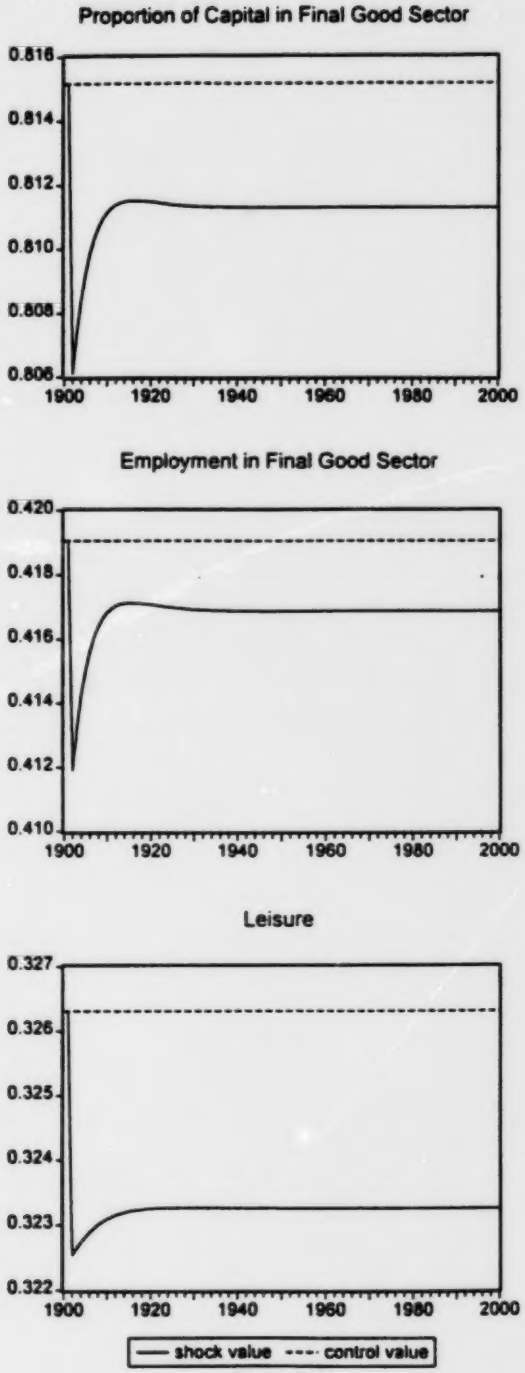
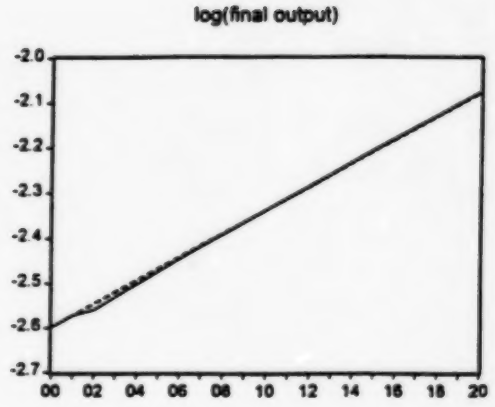
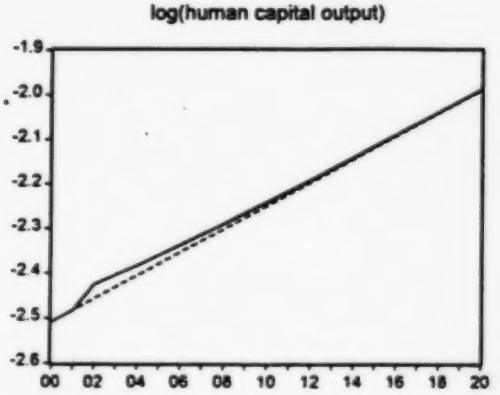

Proportion of Capital in Final Good Sector
0.816
0.814
0.812
0.810
0.808
0.806
1900
1920
1940
1960
1980
2000
Employment in Final Good Sector
0.420
0.418
0.416
0.414
0.412
0.410
1900
1920
1940
1960
1980
2000
Leisure
0.327
0.326
0.325
0.324
0.323
0.322
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 9(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

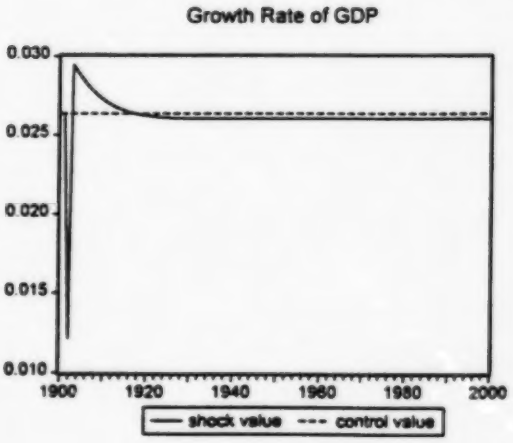

Growth Rate of GDP
0.030
0.025
0.020
0.015
0.010
1900 1920 1940 1960 1980 2000
shock value
control value

Figure 9(B)

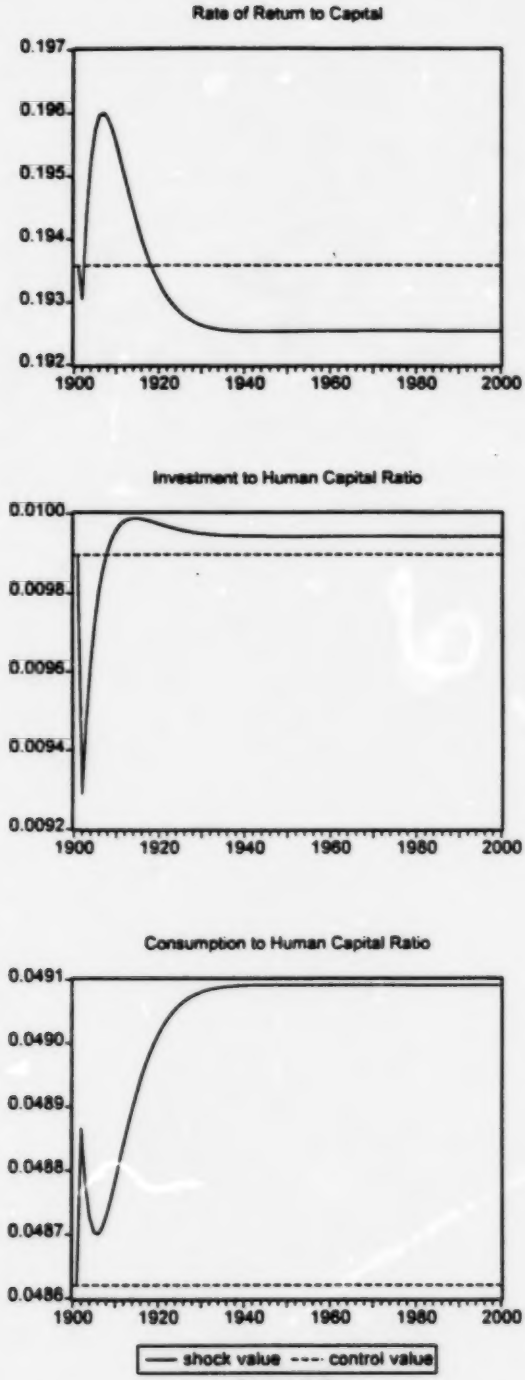

Rate of Return to Capital
0.197
0.196
0.195
0.194
0.193
0.192
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0100
0.0098
0.0096
0.0094
0.0092
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0491
0.0490
0.0489
0.0488
0.0487
0.0486
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 9(C)

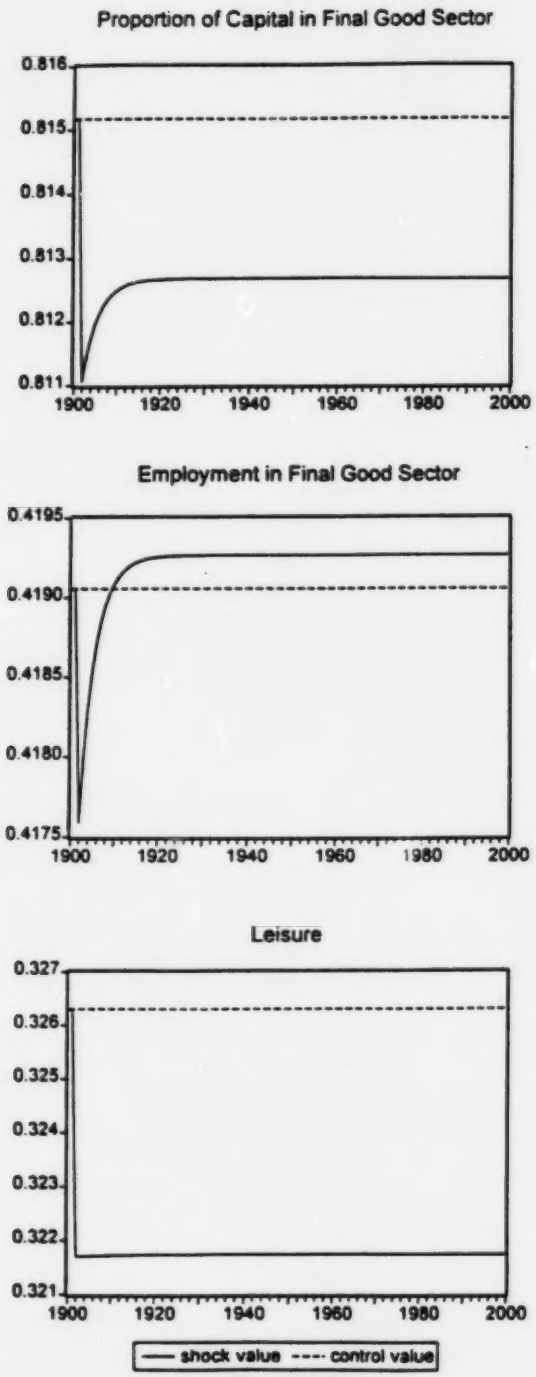
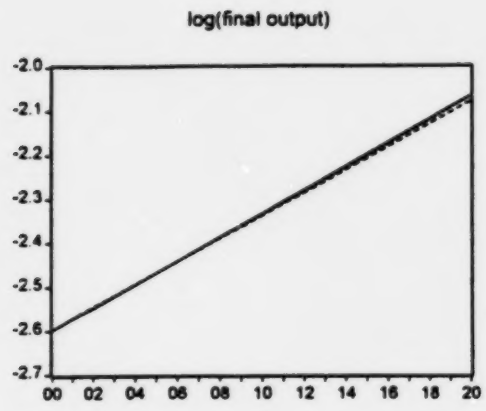
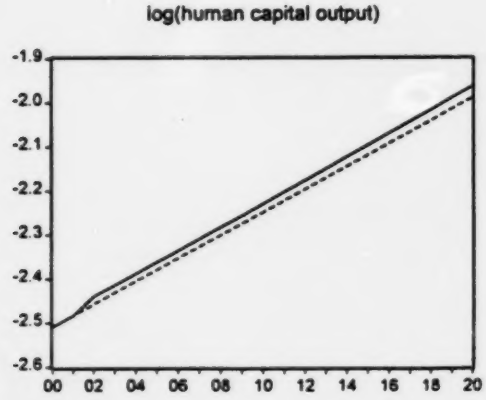

Proportion of Capital in Final Good Sector
0.816
0.815
0.814
0.813
0.812
0.811
1900
1920
1940
1960
1980
2000
Employment in Final Good Sector
0.4195
0.4190
0.4185
0.4180
0.4175
1900
1920
1940
1960
1980
2000
Leisure
0.327
0.326
0.325
0.324
0.323
0.322
0.321
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 10(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

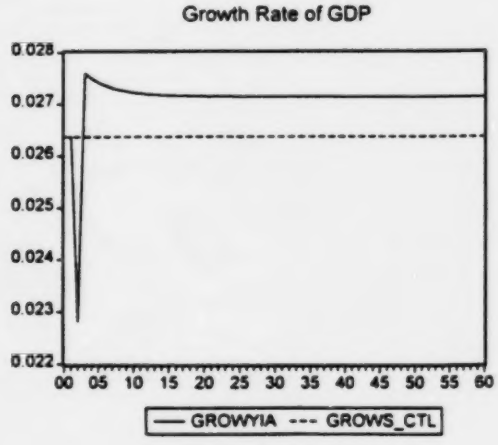
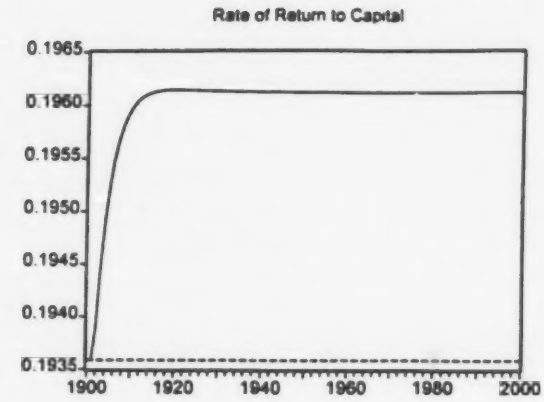
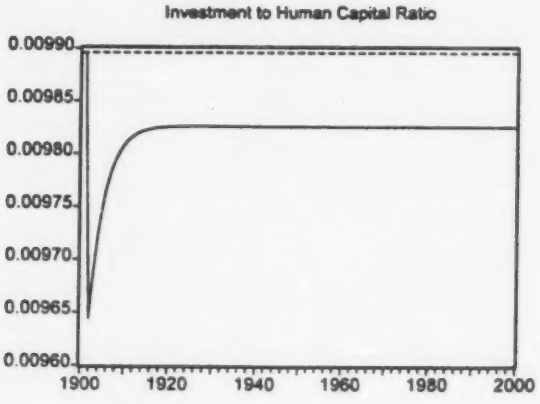

Growth Rate of GDP
0.028
0.027
0.026
0.025
0.024
0.023
0.022
00 05 10 15 20 25 30 35 40 45 50 55 60
GROWYIA
GROWS_CTL

Figure 10(B)


Rate of Return to Capital
0.1965
0.1960
0.1955
0.1950
0.1945
0.1940
0.1935
1900
1920
1940
1960
1980
2000


Investment to Human Capital Ratio
0.00990
0.00985
0.00980
0.00975
0.00970
0.00965
0.00960
1900
1920
1940
1960
1980
2000

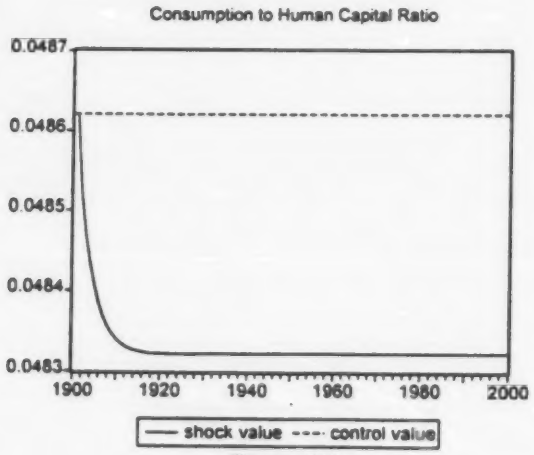
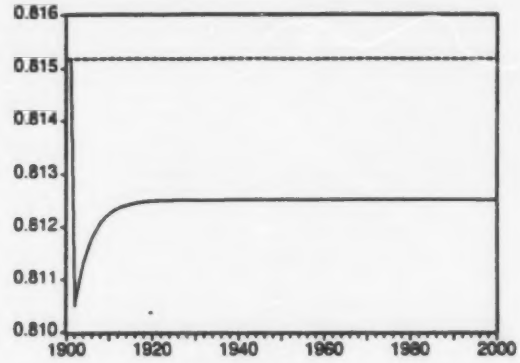
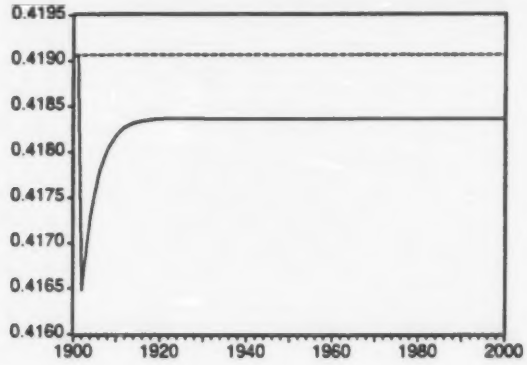

Consumption to Human Capital Ratio
0.0487
0.0486
0.0485
0.0484
0.0483
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 10(C)

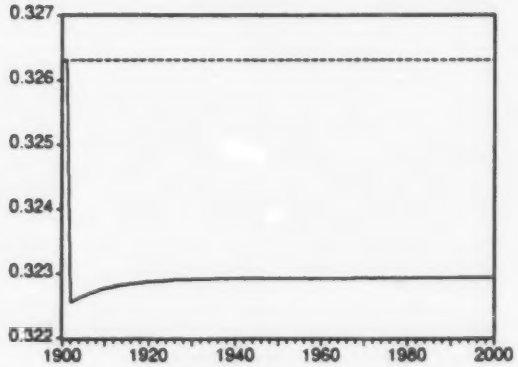
Proportion of Capital in Final Good Sector



Employment in Final Good Sector

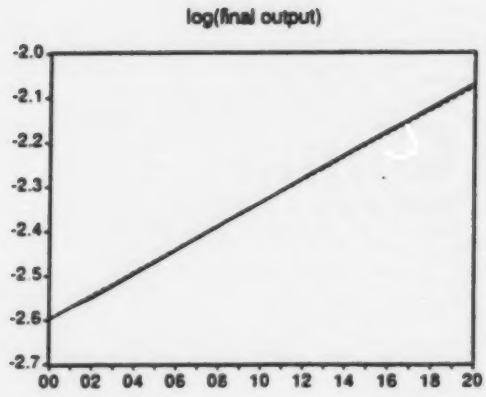


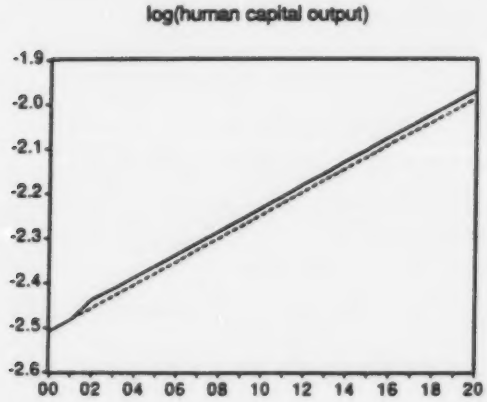
Leisure



shock value ---- control value

Figure 11(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

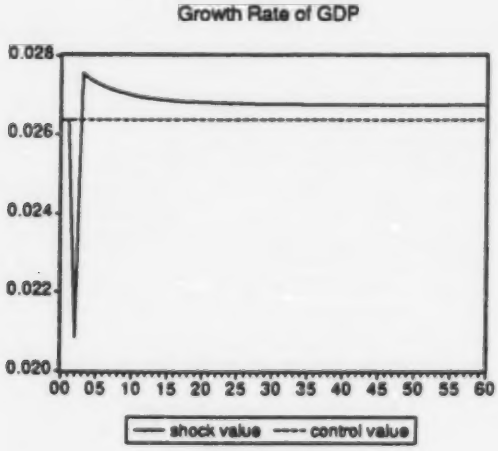

Growth Rate of GDP
0.028
0.026
0.024
0.022
0.020
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 11(B)

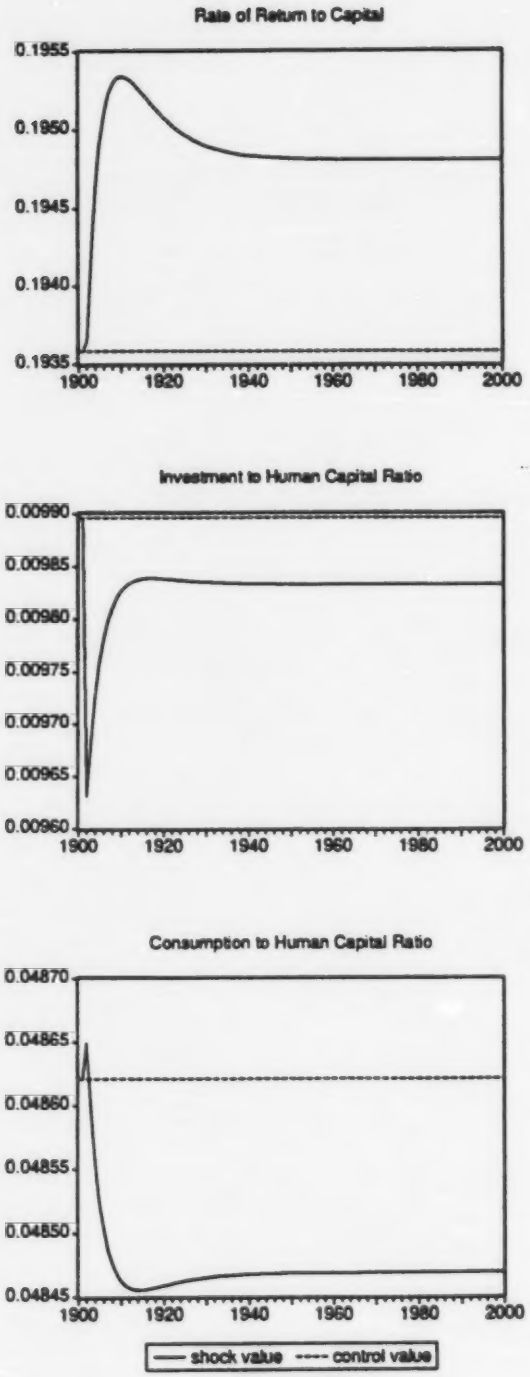

Rate of Return to Capital
0.1955
0.1950
0.1945
0.1940
0.1935
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.00990
0.00985
0.00980
0.00975
0.00970
0.00965
0.00960
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.04870
0.04865
0.04860
0.04855
0.04850
0.04845
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 11(C)

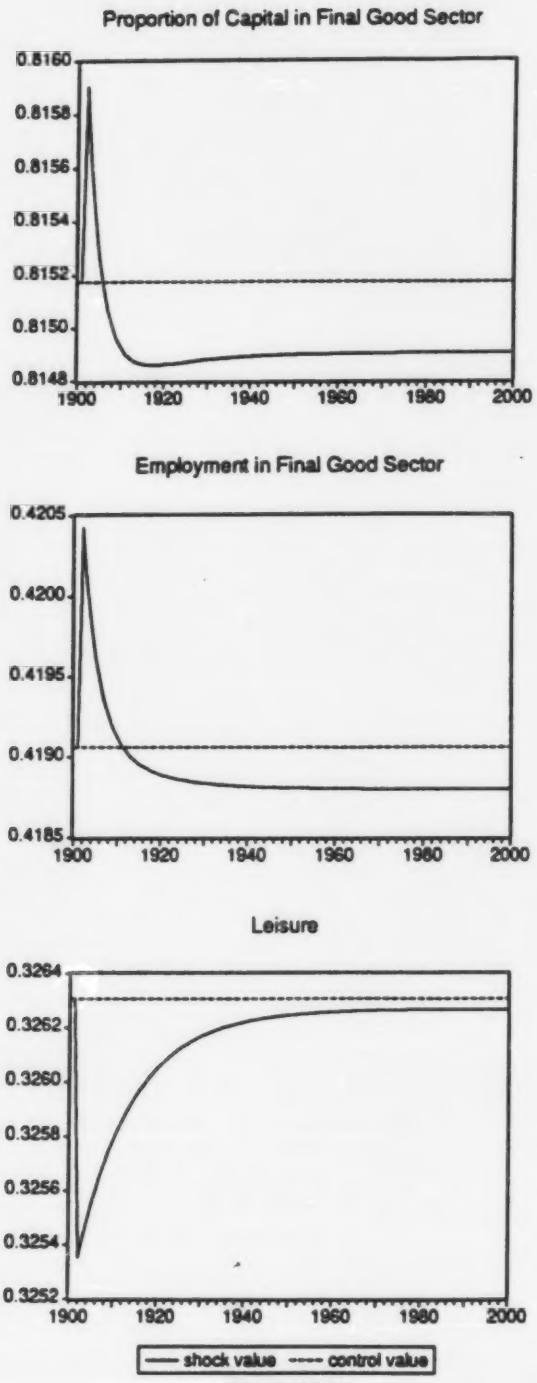
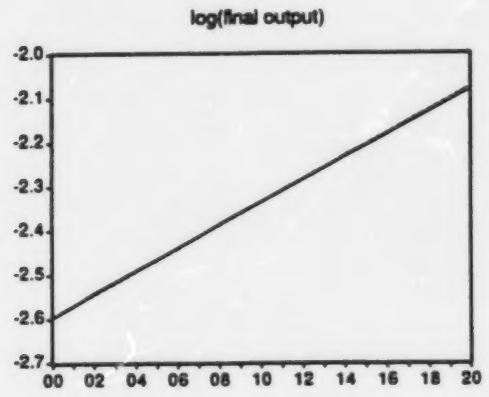
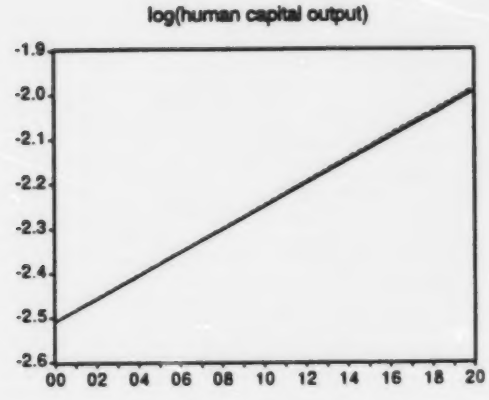

Proportion of Capital in Final Good Sector
0.8160
0.8158
0.8156
0.8154
0.8152
0.8150
0.8148
1900
1920
1940
1960
1980
2000
Employment in Final Good Sector
0.4205
0.4200
0.4195
0.4190
0.4185
1900
1920
1940
1960
1980
2000
Leisure
0.3264
0.3262
0.3260
0.3258
0.3256
0.3254
0.3252
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 12(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

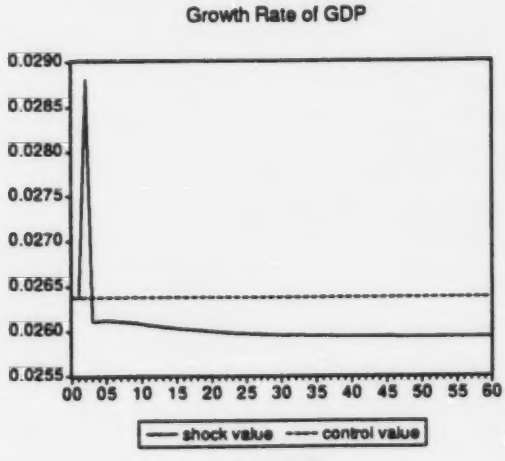

Growth Rate of GDP
0.0290
0.0285
0.0280
0.0275
0.0270
0.0265
0.0260
0.0255
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 12(B)

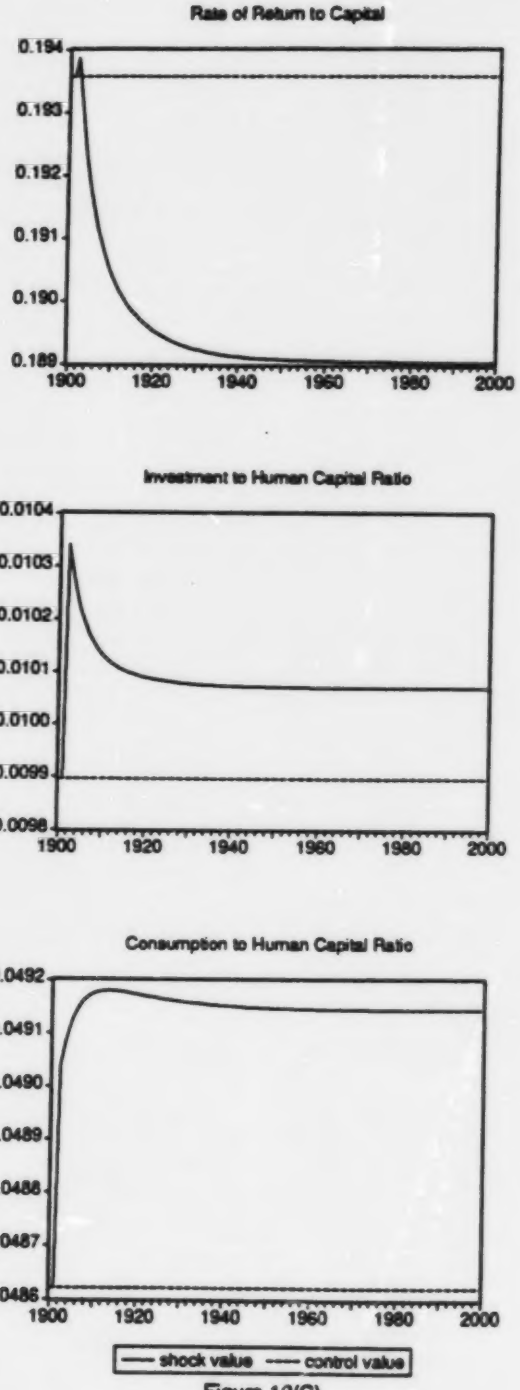
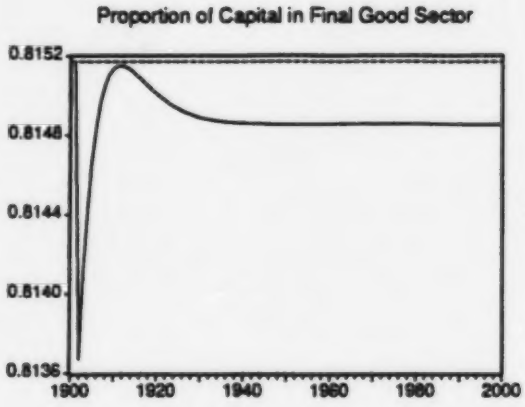
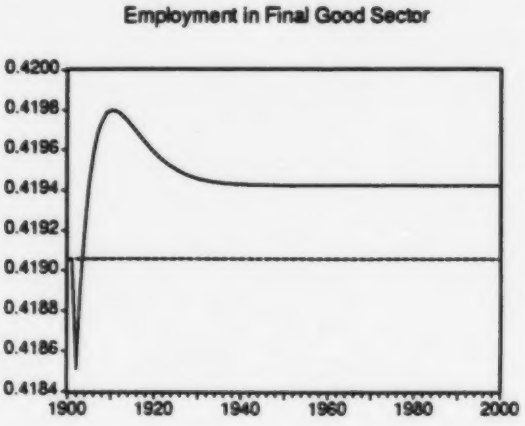

Rate of Return to Capital
0.194
0.193
0.192
0.191
0.190
0.189
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0104
0.0103
0.0102
0.0101
0.0100
0.0099
0.0098
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0492
0.0491
0.0490
0.0489
0.0488
0.0487
0.0486
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 12(C)


Proportion of Capital in Final Good Sector
0.8152
0.8148
0.8144
0.8140
0.8136
1900
1920
1940
1960
1980
2000


Employment in Final Good Sector
0.4200
0.4198
0.4196
0.4194
0.4192
0.4190
0.4188
0.4186
0.4184
1900
1920
1940
1960
1980
2000

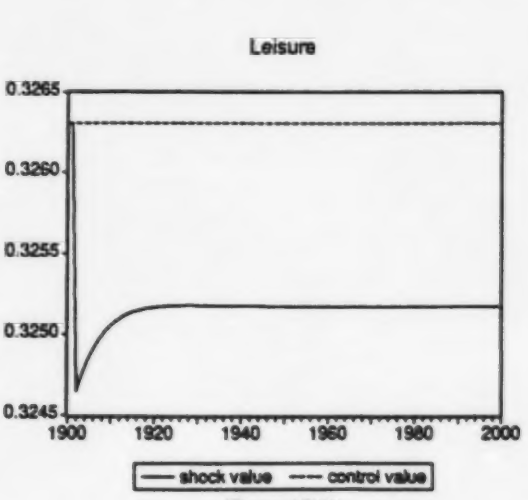
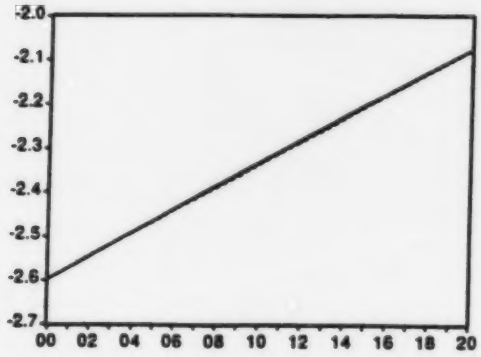
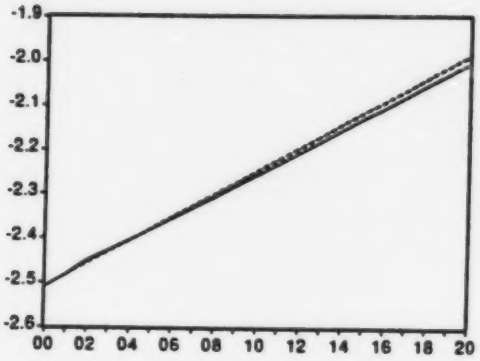

Leisure
0.3265
0.3260
0.3255
0.3250
0.3245
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 13(A)

log(final output)



log(human capital output



Leisure

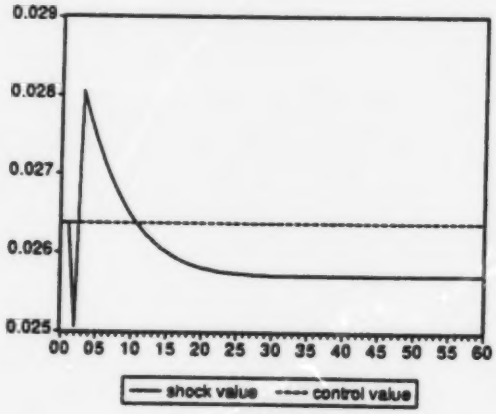


Figure 13(B)

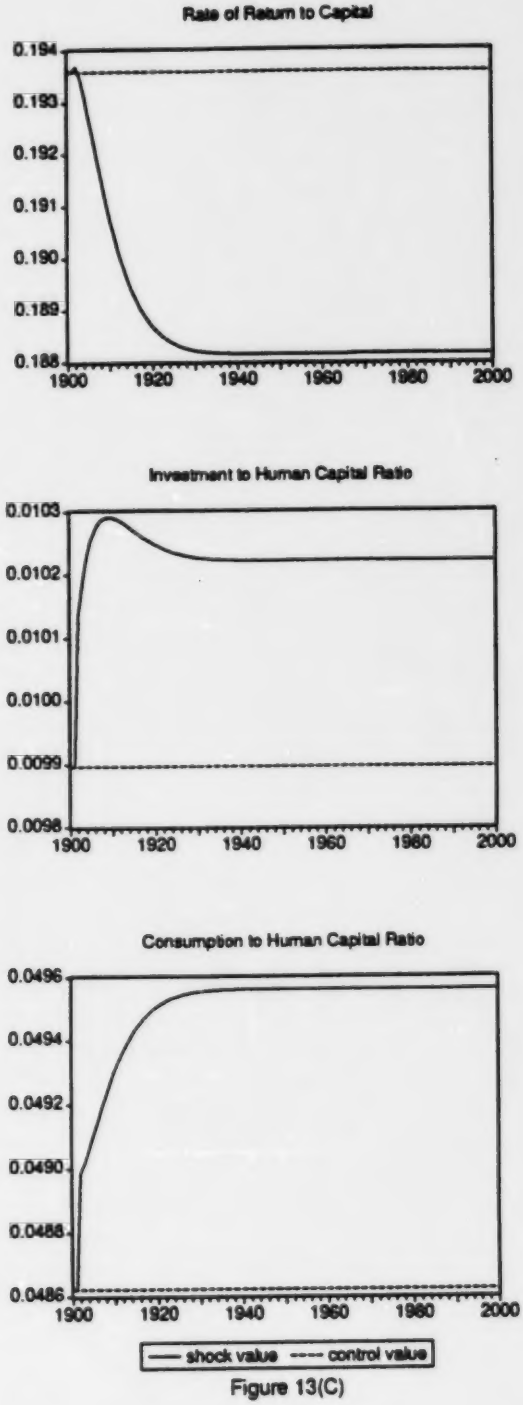
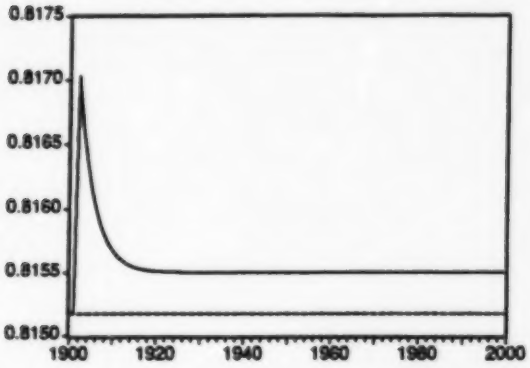
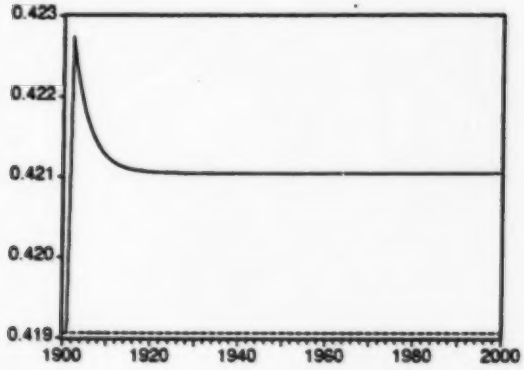

Rate of Return to Capital
0.194
0.193
0.192
0.191
0.190
0.189
0.188
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0103
0.0102
0.0101
0.0100
0.0099
0.0098
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0496
0.0494
0.0492
0.0490
0.0488
0.0486
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 13(C)

Proportion of Capital in Final Good Sector



Employment in Final Good Sector



Leisure

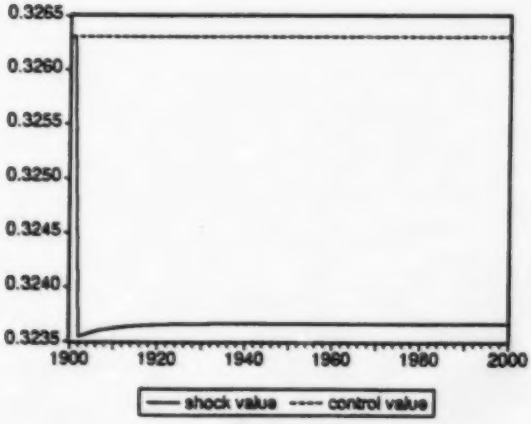
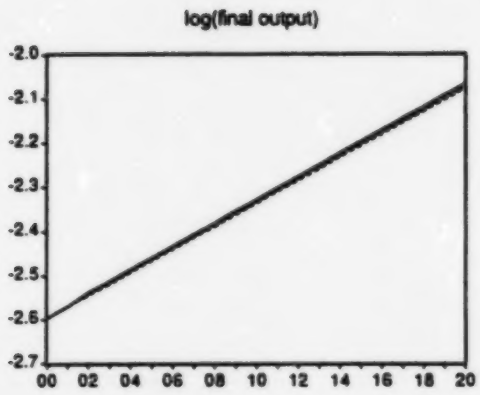
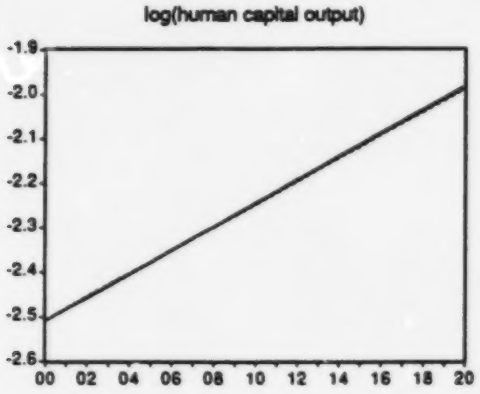


Figure 14(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

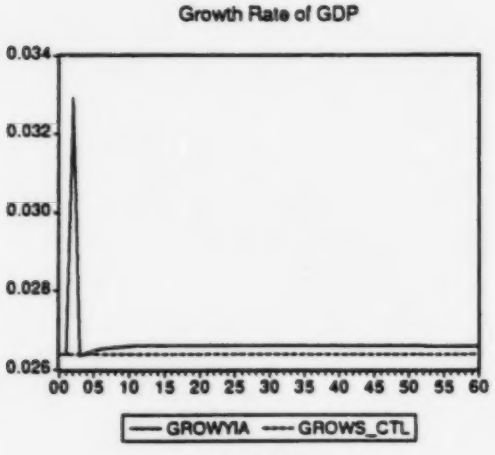

Growth Rate of GDP
0.034
0.032
0.030
0.028
0.026
00 05 10 15 20 25 30 35 40 45 50 55 60
GROWYIA
GROWS_CTL

Figure 14(B)

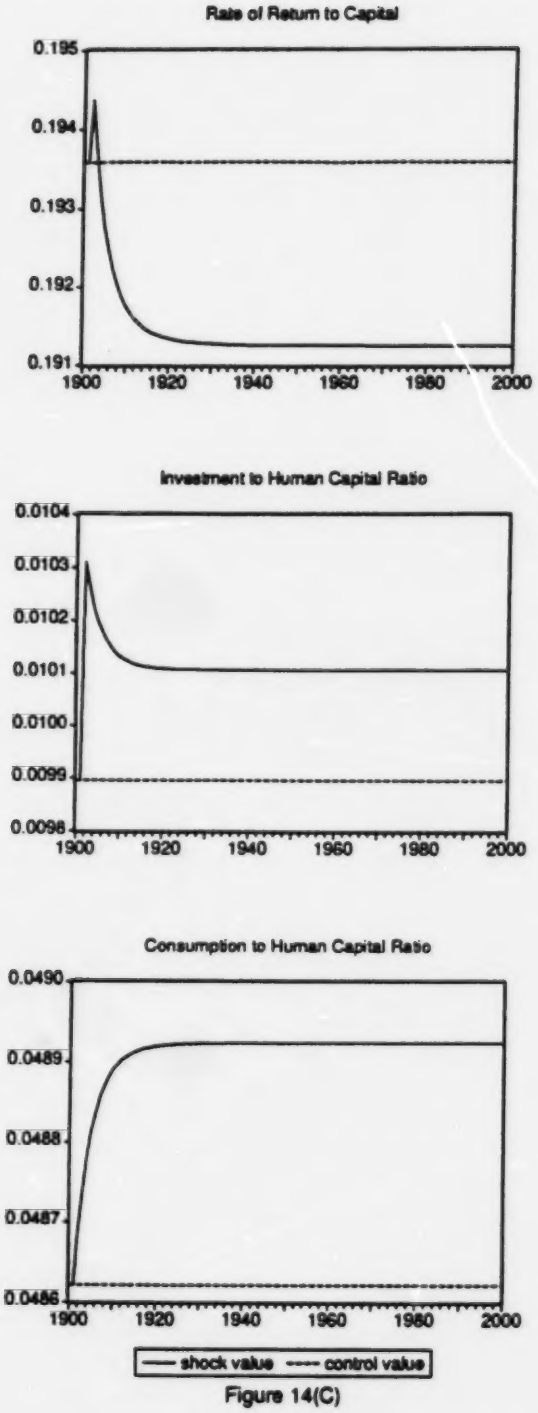
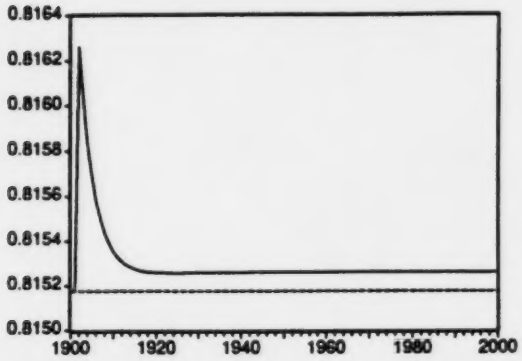
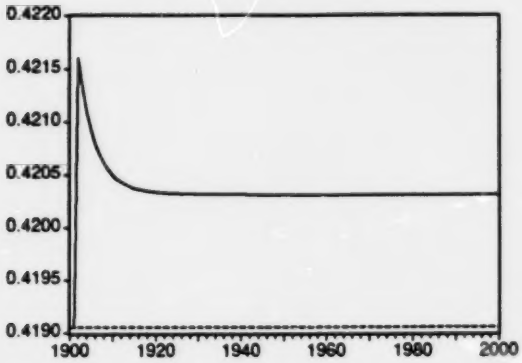

Rate of Return to Capital
0.195
0.194
0.193
0.192
0.191
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0104
0.0103
0.0102
0.0101
0.0100
0.0099
0.0098
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0490
0.0489
0.0488
0.0487
0.0486
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 14(C)

Propportion of Capital in Final Good Sector



Employment in Final Good Sector



Leisure

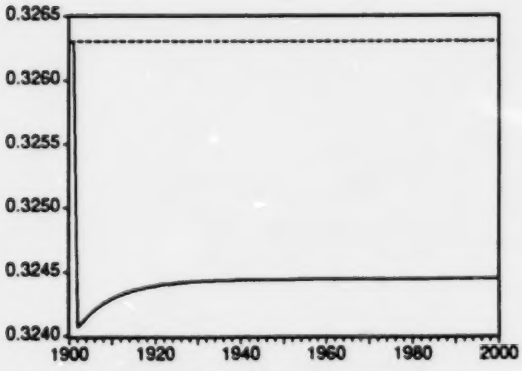
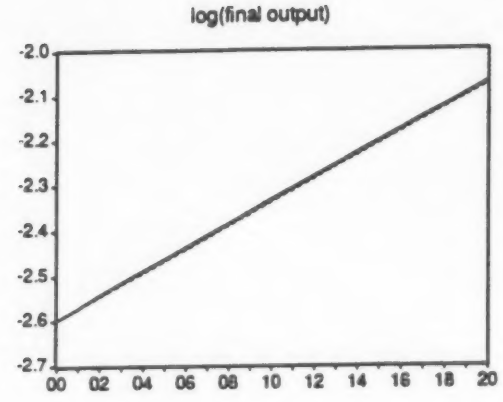
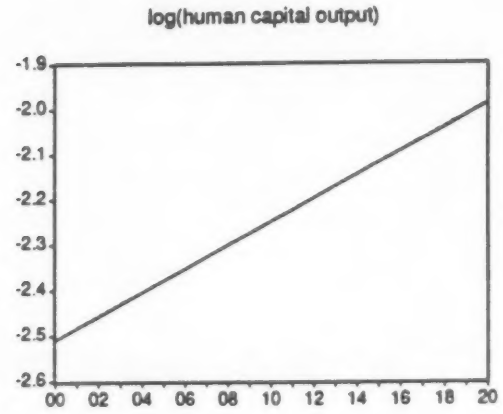


Figure 15(A)


log(final output)
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
-2.7
00 02 04 06 08 10 12 14 16 18 20


log(human capital output)
-1.9
-2.0
-2.1
-2.2
-2.3
-2.4
-2.5
-2.6
00 02 04 06 08 10 12 14 16 18 20

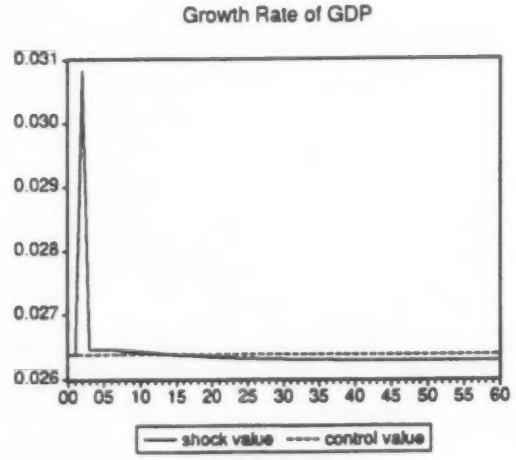

Growth Rate of GDP
0.031
0.030
0.029
0.028
0.027
0.026
00 05 10 15 20 25 30 35 40 45 50 55 60
shock value
control value

Figure 15(B)

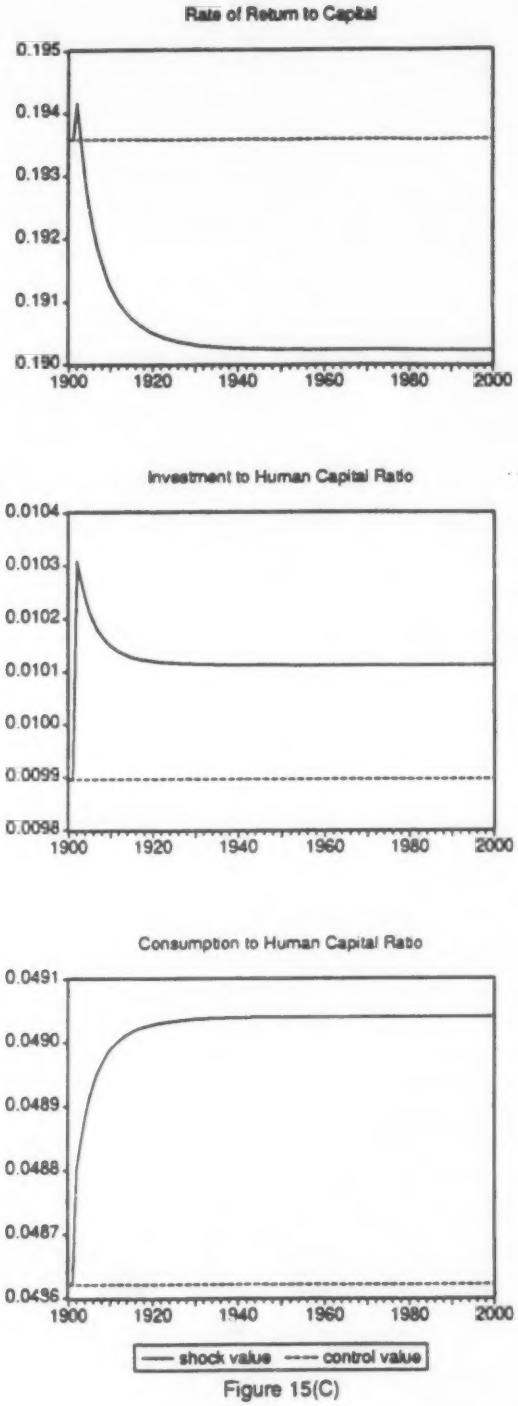

Rate of Return to Capital
0.195
0.194
0.193
0.192
0.191
0.190
1900
1920
1940
1960
1980
2000
Investment to Human Capital Ratio
0.0104
0.0103
0.0102
0.0101
0.0100
0.0099
0.0098
1900
1920
1940
1960
1980
2000
Consumption to Human Capital Ratio
0.0491
0.0490
0.0489
0.0488
0.0487
0.0486
1900
1920
1940
1960
1980
2000
shock value
control value

Figure 15(C)